

# Autonomous Mobile Robots

CHEAT SHEET BY JANIS HUTZ  
ETHZ, FS2026

## 1 Introduction

### 1.1 Probability

**D 1.1** (Sum rule)  $P(X) = \sum P(X, Y) = \sum P(X \cap Y)$

**D 1.2** (Prod)  $P(X, Y) = P(X|Y)P(Y) = P(Y|X)P(X)$

**T 1.3** (Bayes)  $P(Y_i|X) = \frac{P(X|Y_i)P(Y_i)}{\sum_{j=1}^n P(X|Y_j)P(Y_j)}$

**D 1.4** (Cont. Var) Sums become integrals  
e.g.  $\sum_X P(X) = 1$  becomes  $\int p(x) dx = 1$

**D 1.5** (Indep.)  $x, y$  indep. iff  $p(x, y) = p(x)p(y)$

**D 1.6** (Cond. Indep.) iff  $p(x, y|z) = p(x|z)p(y|z)$

**D 1.7**  $E[x] = \int_{-\infty}^{\infty} xp(x) dx$ , also for  $x = f(x)$

**D 1.8**  $\text{Cov}[x] = E[xx^T] - E[x]E[x]^T = \Sigma$

**D 1.9** (Gauss. Dist.)  $x \sim \mathcal{N}(\mu, \Sigma)$  ( $\mu$  mean,  $\Sigma$  cov.),  
PDF:  $p(x) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp(-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu))$

### 1.2 Measurement models

$z = b_C + sM_S\omega + b + n + o$ :  $b_C$  const bias,  $b$  time bias,  $M$  missal.,  $n \sim \mathcal{N}(0, R)$  noise,  $s\omega$  corr. meas.,  $o$  other infl.

### 1.3 Trigonometry

## 2 Locomotion & Kinematics

### 2.1 Positioning

**D 2.1** (Position Vector)  ${}^W t_B = {}^W t_{WB} B$ , Original Frame, End point, Target Frame,  $\sin = s$ ,  $\cos = c$

**D 2.2** (State vector)  $x_R$ :  $x, v$  of rob in  $W$ , pos of sensors

**D 2.3** (Rot. Mat.)  $R_z = \begin{bmatrix} c(\psi) & -s(\psi) & 0 \\ s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_y(\theta) = \begin{bmatrix} c(\psi) & 0 & s(\psi) \\ 0 & 1 & 0 \\ -s(\psi) & 0 & c(\psi) \end{bmatrix} R_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\varphi) & -s(\varphi) \\ 0 & s(\varphi) & c(\varphi) \end{bmatrix}$

**R 2.4** Application:  ${}_W a = R_{WBB} a$

**L 2.5**  $R_{BW} = R_{WB}^{-1} = R_{WB}^T$ ,  $\det(R_{WB}) = 1$  (orth.)

**R 2.6** Cols of  $R_{WB}$  are basis vec. of Frame  $\vec{F}_B$  in  $\vec{F}_W$

**D 2.7** (Euler Angles) Yaw ( $z$ ), Pitch ( $y$ ), Roll ( $x$ ), mult. rotation matrices, e.g.  $R_{EB} = R_z(\psi) \cdot R_y(\theta) \cdot R_x(\varphi)$ , bound.  $[n]^\times = n x^T$  (matrix from vec + arg  $x$ )

**D 2.8** (Rot. Vec)  $\alpha = \alpha n$  ( $n$  normal)

$R(\alpha, n) = I_3 + \sin(\alpha)[n]^\times + (1 - \cos(\alpha))([n]^\times)^2$

**D 2.9** (Quaternions)  $q = q_w + q_x i + q_y j + q_z k$  with  $i^2 = j^2 = k^2 = -1$ ,  $(ij = -ji = k, \text{ same for } jk \text{ and } ki)$

**D 2.10** (Transf. M)  $T_{AB} = \begin{bmatrix} R_{AB} & A t_B \\ 0_{1 \times 3} & 1 \end{bmatrix}$

$T_{BA} = T_{AB}^{-1} = \begin{bmatrix} R_{AB}^T & -R_{AB}^T A t_B \\ 0_{1 \times 3} & 1 \end{bmatrix}$   $T_{AC} = T_{AB} T_{BC}$

### 2.2 Forward Kinematics (FK)

$T_{WB_n}(\theta) = T_{WB_0} T_{B_0 B_1}(\theta_1) \cdots T_{B_{n-1} B_n}(\theta_n)$ .

For 2R system:  ${}^W t_{WE} = \begin{bmatrix} L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \\ L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$

With workspace (pos)  $W$  for  $\theta_1, \theta_2 \in [-\pi, \pi]$

### 2.3 Inverse Kinematics (IK)

Option: Solve Forward Kinematics for angles.

Better: Law of cosine with polar coordinates. Compute angle using cosine rule,

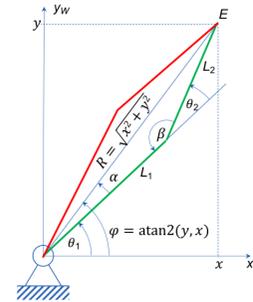
$\theta_1 = \phi \pm \alpha$ ,  $\theta_2 = \pm(\pi - \beta)$

(Positive for Elbow Down, Negative for Elbow Up)

Extension to 6R: 1. Waist: spherical coords (2 sol.)

2. 2 sols from 2R for shoulder + elbow

3. Solve for wrist joints (no influence on pos)



### 2.4 Temporal Models

Often use cont. time n.-lin. system of ODE  $\dot{x} = f_C(x(t), u(t))$ , with measurements  $z(t) = h(x(t))$ . Need linearised (around  $f_C(\bar{x}, \bar{y}) = 0$ , at equilibrium)

### 2.5 Rigid body & IMU kinematics

### 2.6 Rigid Body Dynamics

### 2.7 Wheeled robot Kinematics