

Autonomous Mobile Robots

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1 Introduction

1.1 Probability

D 1.1 (Sum rule) $P(X) = \sum P(X, Y) = \sum P(X \cap Y)$

D 1.2 (Prod) $P(X, Y) = P(X|Y)P(Y) = P(Y|X)P(X)$

T 1.3 (Bayes) $P(Y_i|X) = \frac{P(X|Y_i)P(Y_i)}{\sum_{j=1}^n P(X|Y_j)P(Y_j)}$

D 1.4 (Cont. Var) Sums become integrals
e.g. $\sum_X P(X) = 1$ becomes $\int p(x) dx = 1$

D 1.5 (Indep.) x, y indep. iff $p(x, y) = p(x)p(y)$

D 1.6 (Cond. Indep.) iff $p(x, y|z) = p(x|z)p(y|z)$

D 1.7 $E[x] = \int_{-\infty}^{\infty} xp(x) dx$, also for $x = f(x)$

D 1.8 $\text{Cov}[x] = E[xx^\top] - E[x]E[x]^\top = \Sigma$

D 1.9 (Gauss. Dist.) $x \sim \mathcal{N}(\mu, \Sigma)$ (μ mean, Σ cov.),
PDF: $p(x) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left(-\frac{1}{2}(x - \mu)^\top \Sigma^{-1}(x - \mu)\right)$

1.2 Measurement models

$z = b_C + sM_S\omega + b + n + o$: b_C const bias, b time bias, M missal., $n \sim \mathcal{N}(0, R)$ noise, $s\omega$ corr. meas., o other infl.

1.3 Trigonometry

2 Locomotion & Kinematics

2.1 Positioning

D 2.1 (Position Vector) ${}^W t_B = {}^W t_W B$, Original Frame, End point, Target Frame, $\sin = s$, $\cos = c$

D 2.2 (State vector) x_R : x, v of rob in W , pos of sensors

D 2.3 (Rot. Mat.) $R_z = \begin{bmatrix} c(\psi) & -s(\psi) & 0 \\ s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$R_y(\theta) = \begin{bmatrix} c(\psi) & 0 & s(\psi) \\ 0 & 1 & 0 \\ -s(\psi) & 0 & c(\psi) \end{bmatrix} R_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\psi) & -s(\psi) \\ 0 & s(\psi) & c(\psi) \end{bmatrix}$$

R 2.4 Application: ${}_W a = R_{WBB} a$

L 2.5 $R_{BW} = R_{WB}^{-1} = R_{WB}^\top$, $\det(R_{WB}) = 1$ (orth.)

R 2.6 Cols of R_{WB} are basis vec. of Frame $\vec{\mathcal{F}}_B$ in $\vec{\mathcal{F}}_W$

D 2.7 (Euler Angles) Yaw (z), Pitch (y), Roll (x), mult. rotation matrices, e.g. $R_{EB} = R_z(\psi) \cdot R_y(\theta) \cdot R_x(\varphi)$, bound. $[n]^\times = n x^\top$ (matrix from vec + arg x)

D 2.8 (Rot. Vec) $\alpha = \alpha n$ (n normal)

$R(\alpha, n) = I_3 + \sin(\alpha)[n]^\times + (1 - \cos(\alpha))([n]^\times)^2$

D 2.9 (Quaternions) $q = q_w + q_x i + q_y j + q_z k$ with $i^2 = j^2 = k^2 = -1$, $(ij = -ji = k, \text{ same for } jk \text{ and } ki)$

D 2.10 (Transf. M) $T_{AB} = \begin{bmatrix} R_{AB} & A t_B \\ 0_{1 \times 3} & 1 \end{bmatrix}$

$T_{BA} = T_{AB}^{-1} = \begin{bmatrix} R_{AB}^\top & -R_{AB}^\top A t_B \\ 0_{1 \times 3} & 1 \end{bmatrix}$ $T_{AC} = T_{AB} T_{BC}$

2.2 Forward Kinematics (FK)

$T_{WB_n}(\theta) = T_{WB_0} T_{B_0 B_1}(\theta_1) \cdots T_{B_{n-1} B_n}(\theta_n)$.

For 2R system: ${}_W t_{WE} = \begin{bmatrix} L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \\ L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$

With workspace (pos) W for $\theta_1, \theta_2 \in [-\pi, \pi]$

2.3 Inverse Kinematics (IK)

Option: Solve Forward Kinematics for angles.

Better: Law of cosine with polar coordinates. Compute angle using cosine rule,

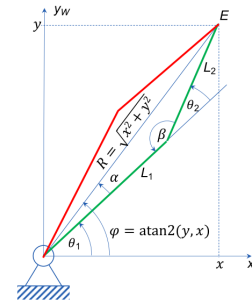
$\theta_1 = \phi \pm \alpha$, $\theta_2 = \pm(\pi - \beta)$

(Positive for Elbow Down, Negative for Elbow Up)

Extension to 6R: 1. Waist: spherical coords (2 sol.)

2. 2 sols from 2R for shoulder + elbow

3. Solve for wrist joints (no influence on pos)



2.4 Temporal Models

Often use cont-time n.-lin. system of ODE $\dot{x} = f_C(x(t), u(t))$, with measurements $z(t) = h(x(t)) + v(t)$. Need linearised (around $f_C(\bar{x}, \bar{u}) = 0$, at **equilibrium**):

$\delta \dot{x}(t) = f_C(\bar{x}, \bar{u}) + F_C \delta x(t) + G_C \delta u(t) + L_C w(t)$

$\delta z(t) = H \delta x(t) + v(t)$. Herein, H is measurements, F_C system, G input gain, w process noise, v measurement noise, both zero-mean **Gaussian White Noise Process**.

For n.-lin. cont-time system: $\dot{x}(t) = f_C(x(t), u(t), w(t))$

$z(t) = h(x(t)) = v(t)$, linearization is the same

To discretize, integrate from t_{k-1} to t_k : $x_k = f(x_{k-1}, u_k, w_k)$

$z_k = h(x_k) + v_k$, linearised:

$\delta x_k = f(\bar{x}, \bar{u}) + F \delta x_{k-1} + G_k \delta u_k + L_k w_k$; $\delta z_k = H_k \delta x_k$

2.5 Rigid body & IMU kinematics

2.6 Rigid Body Dynamics

2.7 Wheeled robot Kinematics