Analysis II

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October 6, 2025

TITLE PAGE COMING SOON

"*Some funny quote from the lecture still needed*"
- Özlem Imamoglu, 2025

HS2025, ETHZ

Cheat-Sheet based on Lecture notes and Script

https://metaphor.ethz.ch/x/2025/hs/401-0213-16L/sc/script-analysis-II.pdf

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1 Introduction

This Cheat-Sheet does not serve as a replacement for solving exercises and getting familiar with the content. There is no guarantee that the content is 100% accurate, so use at your own risk. If you discover any errors, please open an issue or fix the issue yourself and then open a Pull Request here:

`https://github.com/janishutz/eth-summaries`

This Cheat-Sheet was designed with the HS2025 page limit of 10 A4 pages in mind. Thus, the whole Cheat-Sheet can be printed full-sized, if you exclude the title page, contents and this page. You could also print it as two A5 pages per A4 page and also print the Analysis I summary in the same manner, allowing you to bring both to the exam

2 Differential Equations

2.1 Introduction

Ex 2.1.1: $f'(x) = f(x)$ has only solution $f(x) = ae^x$ for any $a \in \mathbb{R}$; $f' - a = 0$ has only solution $f(x) = \int_{x_0}^{x} a(t)\,dt$

T 2.1.6: Let $F : \mathbb{R}^2 \to \mathbb{R}$ be a differential function of two variables. Let $x_0 \in \mathbb{R}$ and $y_0 \in \mathbb{R}^2$. The Ordinary Differential Equation (ODE) $y' = F(x, y)$ has a unique solution f defined on a "largest" interval I that contains x_0 such that $y_0 = f(x_0)$

2.2 Linear Differential Equations

An ODE is considered linear if and only if the ys are only scaled and not part of powers.
D 2.2.1: *(Linear differential equation of order k)* (order = highest derivative) $y^{(k)} + a_{k-1}y^{(k-1)} + \ldots + a_1y' + a_0y = b$, with a_i and b functions in x. If $b(x) = 0 \ \forall x$, ***homogeneous***, else ***inhomogeneous***
T 2.2.2: For open $I \subseteq \mathbb{R}$ and $k \geq 1$, for lin. ODE over I with cont. a_i we have: ***(1)*** Set $\mathcal{S}$ of $k\times$ diff. sol. $f : I \to \mathbb{C}(\mathbb{R})$ of the eq. is a complex (real) subspace of complex (real)-valued func. over I; ***(2)*** $\dim(\mathcal{S}) = k \ \forall x_0 \in I$ and any $(y_0, \ldots, y_{k-1}) \in \mathbb{C}^k$, exists unique $f \in \mathcal{S}$ s.t. $f(x_0) = y_0, f'(x_0) = y_1, \ldots, f^{(k-1)}(x_0) = y_{k-1}$. If a_i real-valued, same applies, but $\mathbb{C}$ replaced by $\mathbb{R}$. ***(3)*** Let b cont. on I. Exists solution f_0 to inhom. lin. ODE and $\mathcal{S}_b$ is set of funct. $f + f_0$ where $f \in \mathcal{S}$
The solution space $\mathcal{S}$ is spanned by k functions, which thus form a basis of $\mathcal{S}$. If inhomogeneous, $\mathcal{S}$ not vector space.

2.3 Linear differential equations of first order

Finding solution set ***(1)*** Find basis $\{f_1, \ldots, f_k\}$ for $\mathcal{S}_0$ for homogeneous equation (set $b(x) = 0$). ***(2)*** If inhom. find f_p that solves the equation. The set of solutions $\mathcal{S}_b = \{f_h + f_p \mid f_h \in \mathcal{S}_\prime\}$. ***(3)*** If initial conditions, find equations $\in \mathcal{S}_b$ which fulfill conditions using SLE (as always)

P 2.3.1: Solution of $y' + ay = 0$ is of form $f(x) = ze^{-A(x)}$ with A anti-derivative of a

TODO: Improve procedure with notes from session & SPAM

2.4 Linear differential equations with constant coefficients

The coefficients a_i are constant functions of form $a_i(x) = k$ with k constant, where $b(x)$ can be any function.
Homo. Sol. Find *characteristic polynomial* (of form $\lambda^k + a_{k-1}\lambda^{k-1} + \ldots + a_1\lambda + a_0$ for order k lin. ODE with coefficients a_i). Find the roots of polynomial. The solution space is given by $\{x^{v_j}e^{\gamma_i x} \mid v_j \in \mathbb{N}, \gamma_i \in \mathbb{R}\}$ where v_j is the multiplicity of the root γ_i. For $\gamma_i = \alpha + \beta i \in \mathbb{C}$, we have $e^{\alpha x}\cos(\beta x)$, $e^{\alpha x}\sin(\beta x)$.