Data Modelling & Databases

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"*A curry can be written in many different ways*"
- Prof. Dr. Gustavo Alonso, 2026

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Contents

1 Introduction

A note on the quote on the title page: He did not *actually* mean "curry", but of course "query", but he just pronounced it that way.

This summary assumes some basic knowledge about databases. All words written in **bold** should be clickable and should link you to the corresponding term in the glossary. If I did not forget a term that is, of course.

1.1 IMS

Early databases were not relational and they also required the user of the system to know *how the data was stored*. This of course is not ideal, as any update to the data structure also required an update to the code interacting with it. In addition, manual query optimization has to be done.

Non-relational databases commonly have data redundancy, due to non-hierarchical applications being forced into a tree-based model. This causes issues when updating the data, as one needs to update the data in all places where a copy of the data exists.

1.2 Network Model

In a network model, one **record** is fetched at a time, so it essentially traverses the network. However, we again do not have data independence, thus the application again needs to change when the physical data representation changes. In addition, manual query optimization has to happen again, thus, any change to the physical data representation may cause the need to redo the optimization.

1.3 Relational Model

This is the focus of this course

The relational model has the benefit of having data independence: The application doesn't know how the data is stored and represented. The queries stay the same, even if the data representation changes. In addition, the end user does not have to worry about optimization too much. This, of course, only applies to a limited extent.

2 Relational Model, Logic & Algebra

2.1 Relational Model

In the relational model, relations are sets of tuples (similar to **records**). They are denoted $R(f_1 : D_1, \ldots, f_n : D_n)$, or sometimes simply $R(D_1, \ldots, D_n)$. An ***instance*** is a ***set*** of tuples $I_R \subseteq D_1 \times \ldots \times D_n$.

Intuition: You can think of the instance to be the rows in the table.

Important We define relations as ***mathematical sets***. This means that they cannot contain duplicates and the order does not matter. Database engines may elect to do this differently, but this course assumes set semantics for the relational model.

2.1.1 Keys

Definition 2.1.1 **Candidate Key** The minimal set of fields that identify each tuple uniquely

Definition 2.1.2 **Primary Key** A single candidate key, i.e. just a single field. We mark the primary key using underlining in visual **schemas**. Formally, we write $R(\underline{k : D_k}, a : D_a, b : D_b)$ for a given relation, with all valid instances fulfilling

$$I \subseteq D_k \times D_a \times D_b \land \forall (k, a, b), (k', a', b') \in I : k = k' \rightarrow (a, b) = (a', b')$$

i.e. if the key is equal, so is the rest of the tuple (thus they are one and the same[1]).

2.1.2 Queries

We write queries in a relational model using the following set notation (using **SQL** for an actual DB):

$$\{(r, p) \mid r \in \texttt{Supplies} \land p \in \texttt{Parts} \land \texttt{r.pid = p.pid} \land \texttt{p.name = "A"}\}$$

The query language thus processes an entire set at a time and applies set operations over the relations.

[1] Heidrun is her name

2.2 Relational Algebra

2.2.1 Operators

- ***Union*** $\cup$: $x \in R_1 \cup R_2 \Leftrightarrow x \in R_1 \vee x \in R_2$ (All tuples from both sets (Only valid for same schemas))
- ***Difference*** $-$: $x \in R_1 - R_2 \Leftrightarrow x \in R_1 \wedge x \notin R_2$ (Tuples that appear in R_1, but not in R_2)
- ***Intersection*** $\cap$: $x \in R_1 \cap R_2 = R_1 - (R_1 - R_2)$ (Tuples that don't appear in both sets)
- ***Selection*** σ: $x \in \sigma_c(R) \Leftrightarrow x \in R \wedge c(x) = \texttt{true}$, with c a predicate on the passed in set. (Tuples that fulfil the predicate c)
- ***Projection*** Π: $\Pi_{A_1,\dots,A_n}(R)$ (Keep only a subset of the columns, e.g. for columns `name`, `pid`, $\Pi_{\texttt{name}}(R)$ returns only the column `name`)
- ***Cartesian Product*** $\times$: $(x, y) \in R_1 \times R_2 \Leftrightarrow x \in R_1 \wedge y \in R_2$ (Primarily used to express join operations. This however simply joins together the two tables (i.e. similar to expanding terms in maths))
- ***Renaming*** ρ: $\rho_{B_1,\dots,B_n}(R)$ (Change the name of the attributes of R to B_i)
- ***Natural Join*** $\bowtie$: $R_1(A, B) \bowtie R_2(B, C) = \Pi_{A,B,C}(\sigma_{R_1.B=R_2.B}(R_1 \times R_2))$. (Join two relations into a table on a column. Natural join joins on columns with same name in both (or all) relations) Edge cases:
  - No shared attributes $R(A, B, C)$, $S(D, E)$: $R \bowtie S = R \times S$
  - All attributes shared $R(A, B, C)$, $S(A, B, C)$: $R \bowtie S = R \cap S$
- ***Theta Join*** $\bowtie_\theta$: $R_1 \bowtie_\theta R_2 = \sigma_\theta(R_1 \times R_2)$ (Join with custom predicate θ)
- ***Equi-Join*** $\bowtie_{A=B}$: $R_1 \bowtie_{A=B} = \sigma_{A=B}(R_1 \times R_2)$ (Join column A with column B)
- ***Semi-Join*** $\ltimes_C$: $R_1 \ltimes_C R_2 = \Pi_{A_1,\dots,A_n}(R_1 \bowtie_C$, with $R_1(A_1, \dots, A_n)$ and $R_2(B_1, \dots, B_m)$ (Returns columns only from one side if there is a match in the join)
- ***Relational division*** $\div$: $R \div S = \Pi_{R-S}R - \Pi_{R-S}((\Pi_{R-S}R) \times S - R)$. In other words, $R \div S = T$, with T being the *largest* relation such that $S \times T \subseteq R$. (Find all rows that do not fulfil a condition)

Semi-Join Reduction A useful trick with semi-joins. It is especially useful in distributed DB, where R and S are on different machines. Suppose we want to Join $R(A, B)$ with $S(B, C)$ on B, then we can do the following $R \bowtie S = (R \ltimes \Pi_B S) \bowtie S$.

The idea is to send the column on which the relations are to be joined from the second system to the first, there execute a semi-join (thus only returning the contents of R that matched with contents of S), then sending only that data to the second machine to finish the full join operation.

Of course, in the real world, users of DB systems don't write relational algebra, as in this case, the order of operations matters for performance, whereas we want database systems to figure this out by themselves.

3 SQL

SQL, or unabbreviated, Structured Query Language, is a **declarative** programming language, used to describe and operate on databases.

This is is the point to mention xkcd standards comic, because, as is the case with almost any standard, people deviate from it and new standards form.

Even though *most* **SQL** databases support the standard SQL operations, many have added a lot of features on top.

SQL is split up into the following parts:

- Data Definition Language, used to create, modify and delete schemas.
- Data Manipulation Language, used to insert, update and delete data
- Query Language, used to retrieve data

3.1 Data Definition Language (DDL)

The DDL is used to describe the schema of relations, in **SQL** called tables. For that, we provide a table name, the columns and their data types.

SQL supports the following data types (plus more):

- `CHAR(n)`
- `VARCHAR(n)`
- `INT`
- `BIGINT`
- `SMALLINT`
- `FLOAT`
- `BLOB(n)` (for large binaries)
- `DATETIME`
- `MONEY`

3.1.1 Creating Tables

Use the DDL `CREATE TABLE` keyword to create a table. We can set default values for certain attributes, or can add constraints to them.

Since the primary key must be unique for each entry, it may be useful to configure the primary key attribute with the following constraints `PrimaryKeyField INT NOT NULL AUTO_INCREMENT`

File: code/sql/ddl/create.sql

```sql
CREATE TABLE TableName (
    Attribute integer,
    OtherAttribute varchar (30),
    NextAttribute character (2) default "AP", -- default value, if unset on insert
    PRIMARY KEY (Attribute) -- primary key, as in RA
);
```

3.1.2 Updating / Altering Tables

Use the DDL `ALTER TABLE` keyword to update a table's schema. Be aware that if we ADD a column to the schema, unless a default value is provided, the column is set to `NULL` (see 3.3.4)

File: code/sql/ddl/alter.sql

```sql
ALTER TABLE TableName ADD COLUMN (col integer);
ALTER TABLE TableName ADD COLUMN (AnotherColumn integer default "");
ALTER TABLE TableName DROP COLUMN AnotherColumn;
```

3.1.3 Deleting Tables

File: code/sql/ddl/delete.sql

```sql
DROP TABLE TableName;
```

3.1.4 Constraints

SQL supports a number of constraints that can be put on the different attributes of a schema.

- `NOT NULL`: Disallows this column to be set to `NULL`, or have a `NULL` value
- `UNIQUE`: The value of each value in this column must be unique, but it can be `NULL`. An arbitrary number of columns can have `NULL` and still be valid

- `PRIMARY KEY`: Sets this column as the primary key. `NOT NULL` and `UNIQUE` is set on it implicitly. Contrary to common intuition, it can be set on multiple columns.
- `CHECK c`: Check that the values fulfil a condition. This condition has the same syntax as for the `WHERE` clause (see 3.3.1)
- `REFERENCES table`: A Foreign Key, used to refer to another tuple from a different relation. It is typically referencing the primary key of the other table.

For `REFERENCES`, there are many options for maintenance, which can be set on creating a reference:

- ***Cascade***: Propagate `UPDATE` and `DELETE`
- ***Restrict***: Prevent deletion of the primary key before the change, causes error
- ***No Action***: Prevent modifications after attempting change, causes error
- ***Set default, Set Default***: Set references to `NULL` or default value

Example 3.1.4

```
CREATE TABLE tab (id integer REFERENCES OtherTable ON DELETE cascade ON UPDATE cascade)
```

3.2 Data Manipulation Language (DML)

The DML is used to insert, update and delete data from the database.

Below some examples showing the use of the common operations. Note that you can use a `WHERE` clause from the query language to filter data for both `DELETE` and `UPDATE` statements.

File: `code/sql/dml.sql`

```sql
INSERT INTO TableName (
    Attribute,
    OtherAttribute
) VALUES ( 1000, 'Value' );

DELETE FROM TableName WHERE Attribute > 50;

UPDATE TableName SET Attribute = Attribute + 1;

-- Import data from a CSV file (this is pgsql specific syntax)
COPY TableName FROM '/data.csv' WITH FORMAT csv;
```

3.3 Query Language

The basic structure of a **SQL** statement is `SELECT I FROM T WHERE C`, which corresponds to $\Pi_{\mathtt{I}}(\sigma_{\mathtt{C}}\mathtt{T})$.

We can set `I = *` if we want to return all columns for a table. To rename, we can set `I = Column as Name, Column2 as Name2`, etc

Theorem 3.3.1 Every SPJR **RA** expression can be written in `SELECT ... FROM ... WHERE ...` form:

Operation	Notation	SQL
Selection	$\sigma_c(R)$	`SELECT * FROM R WHERE c;`
Projection	$\Pi_{A_1,\dots,A_n}R$	`SELECT A1, ..., An FROM R;`
Cartesian Product	$R_1 \times R_2$	`SELECT * FROM (R1, R2);`
Rename	$\rho_{a,b,c}R$	`SELECT A as a, ..., B as c FROM R;`
Union	$R_1 \cup R_2$	`R1 UNION R2;`
Difference	$R_1 - R_2$	`R1 EXCEPT R2;`
Intersection	$R_1 - R_2$	`R1 INTERSECT R2;`

Since **SQL** implements **bag** and not set semantics, there is a `DISTINCT` keyword, which is used to remove duplicates. It is to be applied in the `SELECT` clause (`I` above), e.g.

```sql
SELECT DISTINCT name FROM Data;
```

For the last three operations, `R1` and `R2` typically are other **SQL** statements, example:

```sql
(SELECT name FROM FirstTable) UNION (SELECT name FROM SecondTable)
```

In addition, to apply *bag semantics*, as opposed to *set semantics* append `ALL` to the expression (e.g. `UNION ALL`)

We can also name the tables, e.g. `T = One o, Two t`, and then access columns from a specific table using `I = o.Col, b.Col`, or the like.

3.3.1 Basic operators

3.3.1.1 Logical Operators

The following **SQL** logical operators are available (may not be exhaustive for all systems, `cop` is a comparison operator):

Operator	Description
`P1 AND P2`	TRUE if `P1` and `P2` both are true
`P1 OR P2`	TRUE if one of `P1` or `P2`, or both are true
`C cop ALL query`	TRUE if all values `x` returned in **subquery** `query` fulfil `C cop x`
`C cop ANY query`	TRUE if one (or more) values `x` returned in **subquery** `query` fulfil `C cop x`. `SOME` is logically equivalent
`C BETWEEN low AND high`	TRUE if numerical, text or date value of `C` is between `low` and `high`. Both ends inclusive, negatable
`C IN list`	TRUE if value of `C` is in the provided `list`. List format: `(val, val2)`. List can also be a **subquery**, negatable
`C LIKE R`	TRUE if value of `C` matches regex-like pattern `R`. (% matches any number of chars, _ matches a single, arbitray character)
`EXISTS query`	TRUE if **subquery** `query` retuns at least one row

3.3.1.2 Comparison & Arithmetic Operators

Comparison Operators $<, <=, =, >=, >$ are defined as usually, $<>$ is defined as not equal.

Arithmetic Operators $+, -, *, /, \%$ are defined as usual.

We can use the arithmetic operators like this:

```
SELECT Value * 1.1 FROM Data;
```

or in `WHERE` clauses like this:

```
SELECT Value FROM (Data a, Data b) WHERE a.Value = b.Value - 1;
```

Remark 3.3.2 You may have noticed that in the above query, the same table was used twice. This is referred to as a ***Self-Join*** and can come in handy when comparing values in a single table.

3.3.2 Join

Specifying two (or more) tables in the `FROM` clause will perform a cartesian product, e.g.

```
SELECT * FROM (Table1, Table2);
```

returns all rows of the tables, next to each other.

Here, naming the tables in the `FROM` clause may come in handy, to not write out the entire name of the tables:

```
SELECT * FROM (Table1 t, Table2 s) WHERE t.id = s.id;
```

```
SELECT * FROM (Table1, Table2) WHERE Table1.id = Table2.id;
```

Both of the above statements produce the same result.

3.3.2.1 Nested Queries

To not have the cartesian product executed in the `FROM` clause, or to get a much more complex starting table, we can use a **subquery**. These are parenthesized normal SQL queries.

We can create an alias for them using an `AS name` clause after the parenthesis as follows

```
SELECT r.name FROM (SELECT * FROM Table) AS r;
```

or alternatively, using a `WITH` clause (replace the dummy queries with real queries):

```
WITH CteName (SELECT * FROM Table) SELECT name FROM CteName;
```

3.3.2.2 Join Operations

If we don't specify the kind of join, an `INNER JOIN` is executed. The following three queries are equivalent

```
SELECT * FROM Student, Tests WHERE Student.PersNr = Tests.PersNr;
```

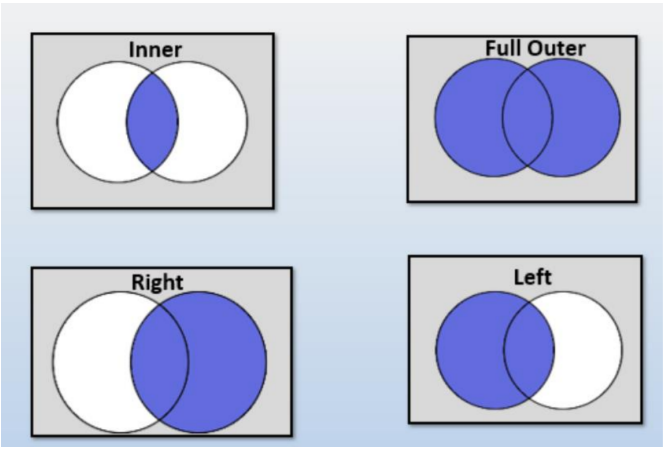
```
SELECT * FROM Student JOIN Tests ON Student.PersNr = Tests.PersNr;
```

```
SELECT * FROM Student JOIN Tests USING (PersNr);
```

For the latter of the three, the parenthesis around the column are important, as omitting them is invalid syntax.

Other types of `JOIN` are:

- `NATURAL JOIN` (no `ON` required), joins on columns with same name
- `LEFT/RIGHT/FULL OUTER JOIN` (requires `ON`), also returns non-matching rows from left, right or both sides, respectively



3.3.3 Aggregations, Grouping, Sorting

3.3.3.1 Aggregations

SQL supports (at least) the aggregation functions `SUM`, `MIN`, `MAX`, `AVG`, `COUNT`, whose output should be evident.

Please note that you cannot use the aggregation functions and still output another column, as they produce a single value.

For each of the functions, you can optionally specify `DISTINCT`.

We typically want to specify an `AS` clause, to make the output nicer to read

3.3.3.2 Grouping

Grouping is used to, well, group together the outputs on a column. Using SQL, this is achieved using the `GROUP BY` clause. We specify the columns on which the grouping should occur.

IMPORTANT We can only use aggregates and columns appearing in the `GROUP BY` clause in the `SELECT` clause.

Using a `JOIN` function, it is of course then possible to go back retrieve the other columns.

To apply filtering on the columns, the `HAVING` clause can be used. It applies a comparison to a grouping column or an aggregate as specified in the `SELECT` clause:

```
SELECT COUNT(name) as cnt FROM Students GROUP BY Department HAVING cnt > 1;
```

3.3.3.3 Sorting

We can sort the output of any query using the `ORDER BY column DIRECTION` clause, where `DIRECTION` is to be replaced with `ASC` or `DESC`:

```
SELECT name FROM Students ORDER BY name DESC;
```

3.3.3.4 Limit

We can limit the number of columns to return using the `LIMIT` clause. Example:

```
SELECT * FROM Exams LIMIT 10;
```

3.3.3.5 Example

Below some examples of **SQL** Queries using grouping, aggregations, sorting and grouping

File: code/sql/aggregations.sql

```
SELECT MAX(price) as MAX_PRICE FROM Products; -- Get the most expensive product
SELECT AVG(grade) as AVG_GRADE FROM Exams WHERE id = 0; -- Get the average grade for an exam

-- Aggregation with grouping (Remember, only aggregates and cols in GROUP BY clause can
-- appear in SELECT clause!)
SELECT id, AVG(grade) as "Average Grade", COUNT(grade) as "Count"
FROM ExamResults GROUP BY id; -- Average grade for each exam and number of students taking it

SELECT id, AVG(grade) as avg, COUNT(grade) as cnt FROM ExamResults
GROUP BY id HAVING cnt > 0 ORDER BY avg DESC LIMIT 20;

-- Get exam details for all exams as filtered before
SELECT * FROM (
    SELECT id, AVG(grade) as avg, COUNT(grade) as cnt FROM ExamResults
    GROUP BY id HAVING cnt > 0 ORDER BY avg DESC LIMIT 20
) as filtered JOIN Exams e ON filtered.id = e.id;
```

3.3.4 Null

Whenever we have incomplete information, we can set a column to be `NULL`. If you have a column that has `NULL` values, when using `WHERE` clauses, these values are skipped. *However* if you are not filtering on the column, a `NULL` value is returned.

To check if a column is `NULL`, you can use the `IS` keyword:

```
SELECT * FROM Data WHERE name IS NULL
```

- In `GROUP BY`, all `NULL`s form a single group.
- `COUNT(*)`, also counts rows with `NULL`
- `COUNT(TheColWithNull)` ignores rows with `NULL`
- Most other aggregations ignore `NULL`
- If all rows in an aggregations are `NULL`, so is the result

The `OUTER JOIN` operators may (and likely will) introduce `NULL`

3.3.5 Views

Views are a useful way of abstracting a complex query and is the result of a stored query.

We can create views using the following template:

```
CREATE VIEW name AS query;
```

We can then operate on it as if it were a table.

Common applications for views are privacy / access control, because you can limit access to tables only via certain views for every user; Usability is also often much better due to having less complex queries to write - you are just writing them once.

How a view is evaluated This is quite simple in essence, the `VIEW`'s query is inserted into the query using the view. It then is then optimized as it normally would be, too. Of course, it is likely that more optimization can and has to be done on views.

3.3.5.1 Updatable Views

Updating Views refers to updating values of the tables in the views. This is not straight forward and thus, many restrictions are placed on what kind of views are considered updatable.

In **SQL**, a view is updatable if and only if it involves ***one*** base relation and its key, it does ***not*** involve aggregates, groups or duplicate elimination.

In short, if there is one-to-one mapping between the view and base relation, it is updatable.

4 Theoretical Background

4.1 Functional Dependency

As with relational algebra, we again define a schema to be a relation, e.g. $\mathcal{R}(A : D_A, B : D_B, C : D_C, D : D_D)$, with an instance of it being $R \subseteq D_A \times D_B \times D_C \times D_D$.

Then, we let $\alpha, \beta \subseteq \mathcal{R}$, i.e. they each have a subset of the columns of $\mathcal{R}$.

$\alpha \to \beta$ (β is a functional dependency of α) if and only if $\forall r, s \in R : r.\alpha = s.\alpha \implies r.\beta = s.\beta$

We write $R \models \alpha \to \beta$ if R satisfies $\alpha \to \beta$ semantically, and $R \vdash \alpha \to \beta$ if the same applies syntactically (i.e. we can apply inference rules over Functional Dependencies).

Intuition: This means that there is a function that maps the values of columns α to the values of columns β.

Definition 4.1.1 $\alpha \subseteq \mathbb{R}$ is a **superkey** if and only if $\alpha \to \mathcal{R}$.

Intuition: This means that if we know the values of columns α, we know the value of the rest of the columns in $\mathcal{R}$. It is a superset of some candidate keys (see 2.1.1)

Definition 4.1.2 $\alpha \to \beta$ is minimal if and only if $\forall A \in \alpha : (\alpha - \{A\}) \not\to \beta$. These are denoted $\alpha \to \cdot\ \beta$

4.1.1 Inference

A fundamental result for a given set of Functional Dependencies F is $F \models \alpha \to \beta \Leftrightarrow F \vdash \alpha \to \beta$ (assuming $\vdash$ is defined by Armstrong's Axioms)

The goal is to find new FDs that can are implied from a set of FDs F on scheme $\mathcal{R}$. Let $\alpha \to \beta$ be another FD on $\mathcal{R}$. Then:

- F implies $\alpha \to \beta$, if every relation instance R of $\mathcal{R}$ that satisfies all FDs in F also satisfies $\alpha \to \beta$.
- The ***closure*** $F+$ of F is the set of all FDs implied by F.
- Let G be another set of FDs on scheme $\mathcal{R}$. F and G are ***equivalent*** ($F \equiv G$) if $F \models G$ and $G \models F$

4.1.2 Armstrong's Axioms

- ***Reflexivity*** $\alpha \subseteq \beta \implies \beta \to \alpha$, special case: $\mathcal{R} \to \alpha$ (referred to as *trivial FDs*)
- ***Augmentation*** $\alpha \to \beta \implies \alpha\gamma \to \beta\gamma$, with $\alpha\gamma = \alpha \cup \gamma$
- ***Transitivity*** $\alpha \to \beta \land \beta \to \gamma \implies \alpha \to \gamma$

These three axioms are all complete and sound. All other possible FDs can be implied from these axioms.

- F ***derives*** $f = \alpha \to \beta$, denoted $F \vdash f$, if there is a derivation for f using only Armstrong's axioms.
- F ***implies*** f, denoted $F \models f$, if for every relation instance R of $\mathcal{R}$ that satisfies all FD in F also satisfies f.

Definition 4.1.3 **Soundness** $F \vdash f \to \implies F \models f$

Definition 4.1.4 **Completeness** $F \models f \implies F \vdash f$

Definition 4.1.5 **Closure** of α with respect to F α^+ is the set of all attributes $y \in \mathcal{R}$ such that $\alpha \to y$ can be derived from F using Armstrong's axioms:

$$\alpha^+ = \{y \in \mathcal{R} \mid F \vdash \alpha \to y\} \qquad F \vdash \alpha \to \beta \Leftrightarrow \beta \subseteq \alpha^+$$

4.1.3 Other rules

- ***Union of FDs*** $\alpha \to \beta \land \alpha \to \gamma \implies \alpha \to \beta\gamma$
- ***Decomposition*** $\alpha \to \beta\gamma \implies \alpha \to \beta \land \alpha \to \gamma$
- ***Pseudo Transitivity*** $\alpha \to \beta \land \beta\gamma \to \theta \implies \alpha\gamma \to \theta$

Algorithm 1 Finding Closure

```
1 procedure FindClosure(F)
2     α⁺ ← α
3     repeat
4         α⁺_old ← α⁺
5         for each FD β → γ ∈ F do
6             if β ⊆ α⁺ then
7                 α⁺ ← α⁺ ∪ γ
8     until α⁺ = α⁺_old
9     return α⁺
```

4.1.4 Closure Algorithm

For the definition of the closure, see above.

Finding a closure α^+ using an algorithm[2]:

We can use that to check the following:

- $F \vdash \alpha \to \gamma$: Calculate α^+ and check $\gamma \in \alpha^+$
- $F \vdash G$: For each $\alpha \to \gamma \in G$, check $F \vdash \alpha \to \gamma$
- F equivalent to G: Check $F \vdash G$ and $G \vdash F$
- $K \subseteq \mathcal{R}$ **superkey** for F: Check $F \vdash K \to \mathcal{R}$:

Minimal Cover **Definition 4.1.6**

A minimal cover (or minimum basis) of F is a set of FDs G that has the following properties:

- $G \equiv F$
- All FDs in G have form $X \to A$, with A being a single attribute
- It is not possible to make G "smaller", i.e. if deleting a FD, $G - \{X \to A\} \not\equiv G$, or $G - \{XA \to B\} + \{X \to B\} \not\equiv G$, for any FD $XA \to B \in G$

Computing the minimum basis:

1. G set of FDs obtained from F by decomposing the right hand sides of each FD to single attribute
2. Remove trivial FDs
3. Remove redundant attributes from LHS of FDs in G (by, for each $X \to Y$ taking each attribute $x \in X$ and, if $X - \{x\} \to Y$ implies $X \to Y$, replacing $X \to Y$ with $X - \{x\} \to Y$)
4. Remove all redundant FDs

4.1.5 Using Functional Dependencies

Functional dependencies help with decomposing relations into several relations that each contain just one concept, since relations combining several concepts are bad.

[2] Yes, I did name the algorithm that. Sorry about that, the pun was too good to skip

4.2 Normal Forms

Definition 4.2.1 **Normal forms** describe the properties of concrete schemas based on functional dependencies in terms of data redundancy and data integrity.

For each of the normal forms, we need to be able to decide:

- whether $\{R, F\}$ satisfies the NF
- given $\{R, F\}$ that satisfies the NF, what are the properties?
- how can we generate new schema R', such that $\{R', F\}$ satisfies the given normal form

4.2.1 First Normal Form

In the first normal form, only atomic domains are allowed, i.e. keys are not allowed to be arrays.

Intuition: This makes the concept of a key easier to define, or well defined

Some **SQL** flavours support arrays. However, the semantics and the support varies

This can come in handy to solve one to many relationships, but what if many different persons have the same elements there? This can cause some issues when updating.

Arrays are very tempting to use if we have an unknown and variable number of different data points for a field.

4.3 Second Normal Form

For a relation to be in 2NF, every non-key attribute needs to minimally dependent on ***every*** key, i.e. no attribute depends on part of a key. If relations are not in 2NF, they may experience insert, update and delete anomalies, which can lead to incorrect, redundant or inconsistent updates of the relations.

However, some relations can still suffer from update and / or delete anomalies

4.3.1 Third Normal Form

A relation R is in 3NF if and only if for all $\alpha \to B$, at least one of the following conditions holds:

- $B \in \alpha$ (then $\alpha \to B$ is a trivial FD)
- B is an attribute of at least one key
- α is a **superkey** of R

Intuition: if $\alpha \to B$ does not satisfy any of these conditions, α is a concept in its own right, thus, the 3NF tries to get rid of "transitive dependencies" (e.g. $A \to B, B \to C$)

The 3NF can still experience update and delete anomalies.

4.3.2 Boyce-Codd Normal Form (BCNF)

R is in BCNF if and only if for all $\alpha \to B$ at least one of the following conditions holds:

- $B \in \alpha$ (thus, $\alpha \to B$ is a trivial FD)
- α is a **superkey** of R

Intuition: In each relation, you only store the same information once.

The following FD are okay in both 3NF and BCNF because both left hand sides are candidate keys for the table.

- $\{\text{PersNr}\} \to$ Name, Level, Room, City, Street, Canton
- $\{\text{Room}\} \to$ PersNr

4.3.3 BCNF Decomposition Algorithm

In words, this algorithm takes as input a schema $\mathcal{R}$ and outputs a list of schemas $\mathcal{L} = \mathcal{R}_1, \ldots, \mathcal{R}_n$, such that $\mathcal{L}$ is a lossless decomposition of $\mathcal{R}$ (i.e. joining them together will perfectly recover the information in $\mathcal{R}$) and each of the $\mathcal{R}_i$ are in BCNF.

Of note is that this does ***not*** preserve the functional dependencies and some data redundancies will still remain, only the ones caused by functional dependencies.

In addition, the order of picking the evil FD matters, so we use heuristics, which pick the one with the largest right hand side.

Algorithm 2 Decomposition algorithm

```
1 procedure DECOMPOSITION(R)
2     result ← {R}
3     while ∃R_i in result such that R_i is not in BCFN do
4         let α → β be the evil FD
5         R_i^1 ← α ∪ β
6         R_i^2 ← R_i − β
7         result ← (result − R_i) ∪ {R_i^1, R_i^2}
8     return result
```

Algorithm 3 Synthesis algorithm

```
1 procedure SYNTHESIS(R)
2     Compute the minimal basis F_c of F
3     for all α → β ∈ F_c do
4         create R_{α∪β}(α ∪ β)
5     If none of the above relations contains a superkey, add a relation with a key
6     Eliminate R_α if there exists R'_α such that α ⊆ α'
```

4.3.4 3NF Synthesis Algorithm

In words, this algorithm takes as input a schema $\mathcal{R}$ and outputs a list of schemas $\mathcal{L} = \mathcal{R}_1, \dots, \mathcal{R}_n$, such that $\mathcal{L}$ is a lossless decomposition of $\mathcal{R}$ and each of the $\mathcal{R}_i$ are in 3NF.

This algorithm preserves all functional dependencies, however does not remove redundancies because it is not BCNF.

5 Systems

5.1 Query Optimization

A lot of performance can be gained in a database system by executing the query in the correct order. For that, a query optimizer exists, which uses a quite large amount of metadata to speed up query execution.

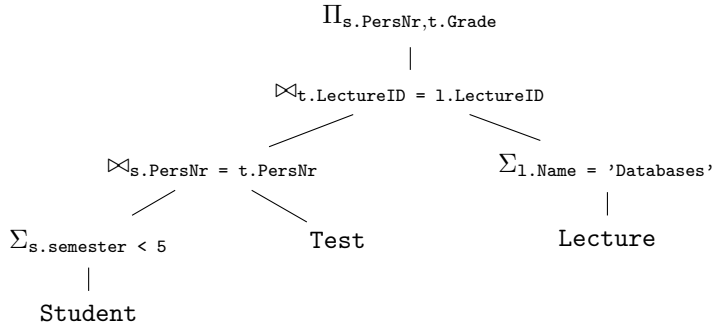
A query optimizer very broadly works as follows:

1. It searches the space of equivalent execution plans. This space tends to be huge, so efficient exploration is needed.
2. Of all the discovered plans, the best is chosen using a cost model, which describes the most efficient plan

5.1.1 Search Space

To create the search space, the optimizer splits a query into a collection of query blocks, where each query block has one `SELECT` and `FROM` clause and *at most* one `WHERE`, `GROUP BY` and `HAVING` clause.

Query plans are typically drawn up as trees, as they are typically easier to understand that way than as both SQL query or in relational algebra notation. In addition, the query parser commonly transforms it into a tree-like data structure, so thinking about query plans in this way is a good habit. For the symbols used here, see section 2.2.



5.1.1.1 Rewriting Rules

Given a logical plan as above, there are multiple ways in which we can construct a physical plan. We can use different join orders, decide when selection happens and specific implementations of operators.

The following rules were discussed in the lectures:

R1 Conjunctive selection operators can be deconstructed into a sequence of individual selections:

$$\sigma_{\theta_1 \wedge \theta_2} = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

This is useful to split selections to push them earlier in the execution plan (reduces the number of elements to process subsequently)

R2 Selection operators are commutative:

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

This is useful to allow to first put the one that e.g. has an index.

R3 Only the last in a sequence of projections is needed, others can be omitted

$$\Pi_{t_1}(\Pi_{t_2}(E)) = \Pi_{t_1}(E)$$

This avoids going over the data twice

R4 Selections can be combined with cartesian products and theta joins

$$\sigma_\theta(E_1 \times E_2) = E_1 \bowtie_\theta E_2 \qquad \sigma_{\theta_1}(E_1 \bowtie_{\theta_2} E_2) = E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$$

This combines the join and selection (thus avoiding having to go over the data twice)

R5 Theta-join operations (and natural joins) are commutative.

$$E_1 \bowtie_\theta E_2 = E_2 \bowtie_\theta E_1$$

This is useful so we ccan choose as the smaller table as the outer table or as the inner one the one with an index.

R6 Natural join operations are associative, so are theta joins (with restrictions)

$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3) \qquad (E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} = E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

This allows for joins to be performed in different orders, allowing us to do the most selective first (fewer rows)

R7 Pushdown selection:

$$\sigma_\theta(E_1 \bowtie E_2) = \sigma_\theta(E_1) \bowtie E_2$$

This allows us to first remove the data that is not needed before doing the more expensive operations

R8 Projections distribute over theta joins (applies if θ involves only attributes from $L_1 \cup L_2$ of course)

$$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1}(E_1) \bowtie_\theta \Pi_{L_2}(E_2)$$

This allows the table width to be reduced before the join to reduce ***data movement***

R9 Union and Intersection are commutative

$$E_1 \cup E_2 = E_2 \cup E_1 \qquad E_1 \cap E_2 = E_2 \cap E_1$$

This allows us to perform one side before the other, depending on resource constraints

R10 Set union and intersection are associative

$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3) \qquad (E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

This allows us to change the order in which operations are done and how the operator tree is executed.

R11 The selection operation distributes over $\cup, \cap$ and $-$

$$\sigma_\theta(E_1 - E_2) = \sigma_\theta(E_1) - \sigma_\theta(E_2) \qquad \sigma_\theta(E_1 \cup E_2) = \sigma_\theta(E_1) \cup \sigma_\theta(E_2) \qquad \sigma_\theta(E_1 \cap E_2) = \sigma_\theta(E_1) \cap \sigma_\theta(E_2)$$

In addition, we have

$$\sigma_\theta(E_1 - E_2) = \sigma_\theta(E_1) - E_2 \qquad \sigma_\theta(E_1 \cap E_2) = \sigma_\theta(E_1) \cap E_2 \qquad \text{not for } \cup$$

This allows us to remove unneeded tuples before performing the next operation, which may be involving comparisons

R12 The projection operation distributes over union

$$\Pi_L(E_1 \cup E_2) = \Pi_L(E_1) \cup \Pi_L(E_2)$$

This allows us to remove columns before the union, requiring less ***data movement***

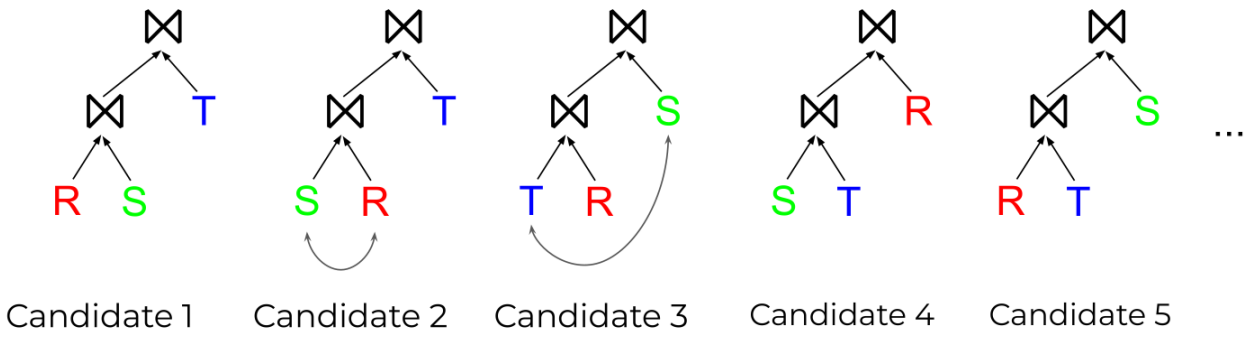


Figure 5.1: Example application of the rewriting rules. (Figure from lecture slides 08, slide 21)

5.1.2 Execution Model

There are a number of ways in which the execution can happen. More details can be found in section 5.2.6

5.1.2.1 Materialization Model

Here, each operator processes its entire input all at once and then emits its output all at once.

This is ideal, if the intermediate results are not much larger than the final results, as very little overhead is created. The efficiency of this approach however is more limited the larger the intermediate results are.

5.1.2.2 Iterator Model

Here, each operator is an iterator, it takes as input a set of streams of tuples (each of which provides a `next()` function) and outputs a stream of tuples, that can in turn again be accessed using a `next()` function. To get a result, we run `root.next()` again and again, until no more data is available to be pulled in.

This is a ***Volcano Mode***, data flows from bottom to top.

This method has the benefits of there being a single interface for all operators, thus we don't (need to) know what the previous operator did. Furthermore, iterators are easy to implement and there are little to no memory overheads and the whole thing can be pipelined, parallelized and distributed.

On the other hand, method calls introduce a large overhead and suffer from poor instruction cache locality.

5.1.2.3 Vectorization Model

This method is similar to the iterator model, but instead of a single tuple, the `next()` invocation returns a batch of tuples.

We can also use vector instructions on the batch of tuple, which reduces overhead by requiring fewer function and instruction calls.

Of course, the limitation here is that the hardware needs to support vector instructions, though nowadays that is hardly an issue, since **AVX** instructions are supported on CPUs as old as the Sandy Bridge Architecture (e.g. Intel Core i7-2600K) for Intel (2011) and Bulldozer (e.g. AMD FX-8100) for AMD (2011).

5.1.3 Cost Model

A cost model is used to ***estimate*** the computation cost of a given physical plan, without actually running it.

Since the ccost of running each operator depends on the input and output size of it, we can easily convert this into cost.

Thus, to estimate the cost, we need to know the runtime of an operator and the number of inputs and outputs. The first of these two aspects is of course very easily attainable, whereas the ***cardinality estimation***, i.e. the estimation of the number of tuples an operator returns, is very hard.

To make this a bit easier, commonly a ***selectivity*** or ***reduction factor*** is used, which tells us how much of the input an operator returns.

Below some reduction factor estimators (we assume that the values are uniformly distributed)

Predicate	Reduction Factor	Predicate Example
`col = val`	`1 / #UniqueValues(col)`	`col = 2`
`col > val`	$\lvert\max(\texttt{col}) - \texttt{val}\rvert \div (\max(\texttt{col}) - \min(\texttt{col}))$	`col > 5`
`col < val`	$\lvert\texttt{val} - \min(\texttt{col})\rvert \div (\max(\texttt{col}) - \min(\texttt{col}))$	`col < 5`
`col1 = col2`	$1 \div \max(\#\texttt{UniqueValues(col1)}, \#\texttt{UniqueValues(col2)})$	`t.name = s.name`

This table shows the overall concept for the cardinality estimation. For each predication, a reduction factor is determined, as each predicate "filters" out some tuples.

Thus, the resulting cardinality is $\max(\#\text{tuples}) \cdot \prod_{r \in RF} r$, where RF is the **bag** of all reduction factors.

In all of the above estimators, we assumed that the values are uniformly distributed. This of course is not always true — there are many cases where data is heavily skewed — in which cases, histograms are used.

Two types of histograms are commonly used:

- ***Equi-Width***: Here, the heights (tuple counts) of the buckets differ, as we fix the values for each bucket. This is useful for identifying hot-spots.
- ***Equi-Depth*** (or Equi-Height): Here, the width differs, because we want all buckets to contain the same number of tuples. The size of a range helps with cardinality estimates
- ***Singleton*** (or Frequency): Plots how often a single item appears in a table. This is very useful to compute the selectivity of queries

To enable quick processing of the cost function, the following metadata, or statistics, are typically stored:

Table statistics

- Number of rows
- Number of blocks
- Average row length

Column statistics

- Number of distinct values (NDV) in columns
- Number of nulls in column
- Data distribution (histogram)

Extended statistics

- Index statistics
- Number of leaf blocks
- Levels
- Clustering factor

System statistics

- I/O performance and utilization
- CPU performance and utilization

5.1.4 Search

We now know how to generate a set of semantically equivalent physical plans and how to estimate the cost for each of them. The question that remains is how to efficiently find the best plan.

Since real-world queries can be very complex, searching for the best physical plan can be hard. There are a few options, but we only covered a simple approach.

We can constrain the search space by only considering ***left-deep join trees***, i.e. trees that have only leafs on the right hand side of each join statement. The rationale behind this is that many of these are fully pipelined plans and no intermediate results have to be written to temporary files. This is the concept `System R`, the first relational database used.

An example for a non-pipelined left-deep join tree is one in volving sort operations. This brings the complexity down to $\mathcal{O}(n!)$, where n is the number of realations in the join.

- Enumerate join orders (different left deep trees)
- Enumerate plans for each operator (hash join, sort merge join, nested loop join, ...)
- Enumerate access methods for each operator (B+-Tree, hash table, sequential scan, ...)

This can be achieved using dynamic programming.

We then find the best 1-relation plan, then the best 2-relation plan by finding ways to join a relation with the best 1-relation plan, etc.

While this is still slow, it's faster than enumerating all possible join trees.

Later, systems started introducing heuristics to reduce the number of choices that must be made in a cost-based fashion.

Heuristic optimization transforms the query-tree using a set of rules typically improve execution performance:

- Perform selection early (reduces the number of tuples to process)
- Perform projection early (reduces the number of attributes to process)
- Perform most restrictive selection and join operations before other similar operations

Some systems only use heuristics, while others combine them with partial cost-based optimization.

5.2 Database Engines

5.2.1 Storage Management

To make database engines easy to use, some abstractions have to happen.

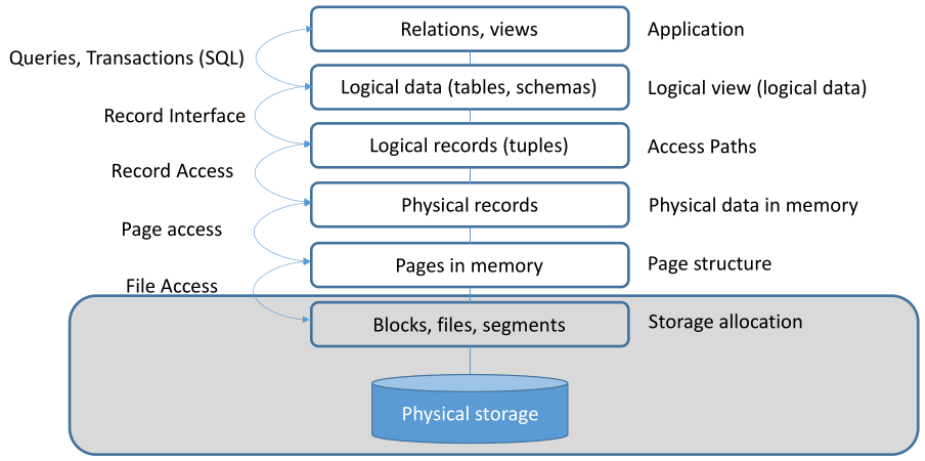


Figure 5.1: Overview of a typical abstraction for a database system (Figure from lecture slides 09, slide 3)

In the design of database engines it is commonly assumed that the data is stored on spinning disks, i.e. HDDs, with a high latency for seeks (random accesses) and that most of the DB is not in memory.

Very modern database systems may assume that a higher percentage of the DB is stored in memory due to the higher quantity of memory available, as well as lower access latencies due to the more readily available SSDs and high-speed NASes.

Some database systems operate similarly to operating systems in certain aspects, in that they manage their own processes and all of the memory to get the absolute maximum performance out of the systems.

5.2.2 Physical to logical storage

To the left, in figure 5.1, the relationship between the different storage abstractions are connected using crow's feet.

Since database tables often include large amounts of data, that would span many OS pages, thus database systems use blocks as opposed to OS pages.

These database blocks are structured as follows:

- ***Header***: Contains the address and type of segment (that being index, table, etc)
- ***Table directory***: Contains the schema of the table stored in the block
- ***Row directory***: Contains pointers to the actual tuples stored in the block. Grows downwards (like the stack)
- ***Free Space***: Well... it's free real estate, isn't it?
- ***Row data***: The tuples stored in the block. Grows upwards (like the heap). The pointers in the row directory point here

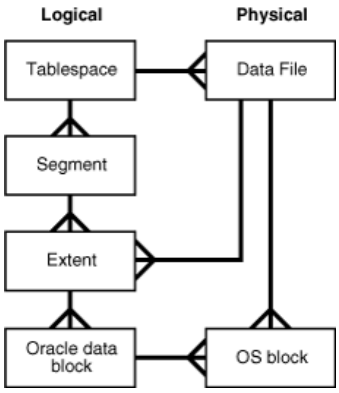


Figure 5.1: Entity relationship diagram, where the crow's feet imply one-to-many relationships (Figure from Oracle Docs)

Typically, database systems allow inserts of new tuples until there is about 20% of free space left. This space is allocated to allow for updates of existing tuples, as they may become larger. Thus, a larger storage footprint is used to prevent having to move all the data to a different location when the data block inevitably fills up, avoiding thrashing on the page.

Some systems even go a step further, only again unlocking a data block for writing if its used percentage falls below 40%.

Of course, as with any storage system, fragmentation becomes an issue. Compaction is only done when the block has enough space for an `INSERT` or `UPDATE`, but the space is not contiguous.

An `UPDATE` may require that a tuple is moved into a different block. In that case, the tuple is inserted into a new block and a pointer is stored in the freed space.

5.2.3 Record Layout

Each record (commonly called tuple) contains a header (which in turn contains validity flags for deletion, visibility information for concurrency control, bit maps for null values and more), and the non-null attributes, or pointer to those attributes.

Relational engines do not have to store the schema for the tuple inside of it because that is set globally for the entire table.

Schema-less systems however have to store that information because it can differ for each record.

5.2.3.1 Optimizations

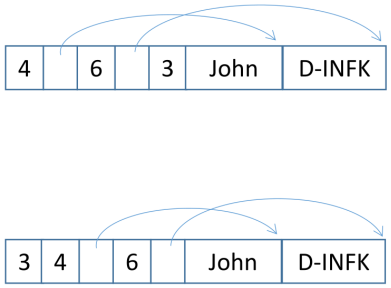


Figure 5.1: Record layouts in memory (Figure from Lecture 09, Slide 19)

It is crucial to store the data in a way that allows quick accesses and updates.

Each variable length attribute also has to store the length, so in a simple system we would simply store them together with the data. This however is not ideal for performance reasons (we have linear access time), thus we keep the fixed size part (attributes and lengths) at the start of the record and move the variable length part to the end, putting pointers after the length attributes that point to the end of the variable length region. Even more performance can be gained by reordering the attributes altogether, moving the variable length attributes to the very end and then applying the above optimization.

5.2.3.2 Data Types

- ***Integers***: Most **DBMS** use the same format as `C`.
- ***Real numbers***: Either in IEEE-754 (for variable precision) or using fixed point representations, which commonly are variable length, by storing all digits, plus where the decimal point is (they are NOT stored as strings)
- ***Strings*** and ***BLOBs***: Store the length and the data
- ***Time***, ***Coordinates***, ***points***, ...: System specific.

For large attributes, commonly instead of storing the whole tuple, only the fixed length attributes of the tuples are stored directly in the block, with pointers to the rest of the attributes, that may also lay outside of the current block. BLOBs may also be *very* large, taking up more than a single block.

5.2.4 Data in the blocks

The pages allocated to a given object (table, index, etc) are managed through lists, which are stored as part of the segment header

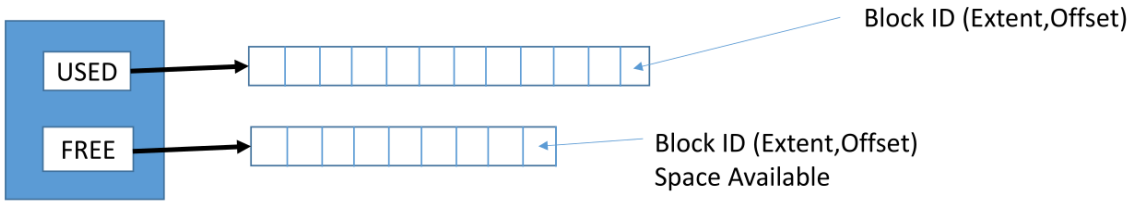


Figure 5.2: Free/used lists in databases (Figure from Lecture 09, Slide 24)

These lists can become a bottleneck, especially for transactions that result in modifications. There are some ways to improve the performance:

- We can use several free lists, so concurrent transactions can find free space in parallel, rather than having conflicts all the time
- Make the traversal of the list faster (sorting, etc)
- Make sure that holes can be found efficiently (store the available space in each page in incremental steps)

5.2.4.1 Slotted Pages

To find tuples within pages, slotted pages are used often. In slotted pages, each page has a header containing checksum, version, transaction visibility, compression information, utilization, etc.

Then, each tuple gets an id (typically a block ID and an offset).

The page maintains a list of the "slots" (each slot is assigned to one tuple and thus the number of tuples in the pages is further constrained by the number of pages available). The offsets, or sometimes pointers, are stored at the beginning of each tuple.

Advantages of slotted pages are:

- storing tuples of different sizes (and variable length tuples)
- Each tuple gets a permanent tuple / record id, that does not change
- When data doesn't change, space is used efficiently
- When data *does* change, we need to be careful

Considerations for when data *does* change:

- If a tuple is modified and becomes larger, the original space is used to store a pointer to the new location of the tuple
- If a tuple is deleted, we just remove the pointer
- If a tuple is modified and becomes smaller, leave the unnecessary space empty
- For insertion, we look for a page with enough free space and compact the page, if needed

The percentage free measure is used to avoid fragmentation caused by pages running out of space to accommodate updates for tuples that become larger. Thus, some space is reserved for growing the tuples, rather than for inserting.

Without this measure, the block can constantly go from `FREE` to `USED` and back and forth.

5.2.5 Row and Column Stores

5.2.5.1 Row Store

Here, as we have assumed thus far, tuples are stored with all their attributes together. This means that accesses to tuples is quick and easy.

This scheme is used by actual systems, but they also have many additional details added to improve management and speed.

A typical ideal application for a row store is Online Transaction Processing (OLTP). This is visible for example with banking applications, shopping carts, etc, where the operations mostly are on a single tuple.

However, for xomplex queries, the row store can become cumbersome.

5.2.5.2 Column Store

This is where the column store comes in, in which the data is stored by columns. This obviously speeds up operations on the columns, as the access is very quick.

Column stores also have the advantage that they present the data exactly in the representation needed for **SIMD** instructions such as **AVX**, which is now widely available on modern CPUs.

Modern database systems have started to adopt column stores to address the memory wall, since Column Stores are a more compact representation and cache lines are likely to bring the data we want to see, thus the cache utilization tends to be higher than with row stores.

Column store databases are today primarily used today for analytical databases and in-memory databases.

An alternative is a hybrid approach, called ***Partition Attributes Across (PAX)*** representation. There, each block contains several tuples, but it is organized as a column store.

5.2.6 Execution Models

We model execution as a tree of operators (as seen previously). At the root, we have the query results, at the leaves we have the tables. Typically, each operator has *at most* two inputs from lower layers (join operators have two, the rest typically has one).

We remember that we can use any subtree of a query as input for the higher parts of the tree, since we can "materialize" the result as a table.

5.2.6.1 Basic Execution Models

Each node (operator) in the tree operates independently, i.e. does input, putput, output.

Each operator can read their input tuples in one go, or one at a time and the same goes for the output. Some operators may also be *blocking*, i.e. no result can be issued until all operations are completed.

5.2.6.2 More advanced models

As a reminder, we already covered some aspects of three execution models in section 5.1.2.

5.2.6.2.1 Iterator Model

Some typical dataflow operators:

- ***Union***: read both sides, issue all tuples (if they have the same schema)
- ***Union without duplicates***: We first union, then we check for duplicates
- ***Select***: We apply a filter function to the tuples
- ***Projection***: For each tuple, we output only the specified attributes
- ***Join***: Trivial nested loop join
- ***Join with index on inner relation***: For all tuples, fetch matchin tuple in S ¡= another operator

Advantages

- Generic interface for all operators
- Easy to implement operators
- Supports buffer management strategies
- No overhead in terms of main memory
- Supports pipelining
- Supports parallelism and distribution

Disadvantages

- High overhead of method calls
- Poor instruction cache locality

5.2.6.3 Pull & Push Mode

Definition 5.2.3 **Pull Mode** Intuitively like dynamic programming: Data is obtained by one operator through a function call to a lower operator. It is good for disk based systems and when data doesn't fit into memory and is also really easy to implement.

Definition 5.2.4 **Push Mode** This is the Pull Mode flipped on its head. While it *can* be faster, there need to be buffers everywhere and it is significantly harder to implement.

5.2.6.4 EXCHANGE operator

The `EXCHANGE` operator is useful for parallel processing, as it simply moves data from one place to another. This allows to parallelize a plan across one or more machines.

This can be implemented as a sort of "driver" on top of an operator, by calling `next()` on the operator a few times, collecting the results and sending them to the other side. The other side uses the data to respond to further next calls to the original operator.

5.3 Indexing

5.3.1 Access methods

Definition 5.3.1 **Access Methods** typically refers to the operations and data structures used for reading data from the buffer cache into the private space of the worker thread

The basic access method is ***sequential scan***, where all pages of the table are read from beginning to end. Indexing is used to avoid having to scan the whole table.

There are many indexing techniques, but we will focus on three indexing techniques in this section, hashing, B+ trees and bitmaps.

Hashing is primarily used to randomize access to the data, finding data based on a key (hash and access concept) and both as an index for tables and used to implement operators.

B+ trees on the other hand are used to order data along the values of one attribute, but require traversal, unless we are looking for consecutive values. They are thus only used as an indexing mechanism.

5.3.2 B+ Trees

5.3.2.1 B-Tree

In B-Trees of order k, each node (except the root) has between $k \div 2$ and k child nodes. The root node has at least two children, unless it is also a leaf and any non-leaf node contains at exactly $k - 1$ keys.

Even if database systems say they use B-Trees, they do in fact use B+ trees.

5.3.2.2 Basics

B+ Trees are B-Trees, but the data (or the pointers to it) is at the leaves only, which are organized as a linked list. They are balanced trees, their inner nodes correspond to blocks and the leaf nodes correspond to blocks. The blocks are organized like slotted pages for variable length data (see Section 5.2.4.1)

The keys in the inner nodes may not correspond to the actual data and are instead used as separators. Typically, leaf nodes contain pointers to the tuples `(value, key)`, where `value` is the actually indexed value and `key` is the pointer to the full data tuple, typically as a row id or tuple id. There are some systems that allow storing the data directly on the leaves.

Some systems create a B+ tree index by default for all tables and index the key. If there is no key, the engine assigns random keys and indexes them.

5.3.2.3 Clustered Indexes

An index orders the table by the attribute it is indexing, but the tuples in the table might not be ordered. This is where ***clustered indexes*** come into play: They force the tuples to be stored in the same order as the index indicates, thus, the table is physically stored in a sorted manner. This is typically done only for the primary key, and automatically done in systems that store data in the leaf nodes directly.

This eliminates random memory accesses caused by looking up ranges, since the tuples are stored in consecutive order, according to the index.

This is most useful for tables that are infrequently updated, since they require maintenance.

5.3.2.4 What to index?

This is a crucial question, as it can massively speed things up, but equally significantly slow things down.

B+ trees can use one or *more* attributes as the key to the index. If we have just one attribute, those are the values. If there are several attributes, it builds a composite key, which is useful when we are looking for combinations of values on certain attributes, or a table is searched by several attributes.

We then compare the keys lexicographically:

$$(a_1, a_2) < (b_1, b_2) \Leftrightarrow (a_1 < b_1) \vee (a_1 = b_1 \wedge a_2 < b_2)$$

Some values can also be left unspecified, but this is typically only done to the ones at the beginning of the key.

5.3.2.5 Composite Index

For a **SQL** statement such as

```
SELECT department_id, last_name, salary
FROM employees WHERE salary > 5000
ORDER BY department_id, last_name;
```

if we assume a composite index on `department_id, last_name, salary`, then the data is sorted by the three attributes in that order.

We can then perform a ***full index scan***, where read the data from the leaves in sorted order and filter on salary. This avoids having to scan the entire table and the results is also already sorted.

5.3.2.6 Non-unique values

A B+ tree index can also be built on attributes containing also non-unique values.

This is a problem, because we cannot find all the duplicates with the basic design. To resolve this, we can repeat the key at the leaf nodes for every duplicated entry, or we can store the key once, but point to a linked list of all matching entries.

If the data is stored in the leaves, we append the tuple ID to know what tuple the entry refers to, because otherwise, they are all the same.

5.3.2.7 Operations

5.3.2.7.1 Direct Lookup

We traverse the tree from the root, then within each node, we use binary search to look for the correct entry. When we reach a leaf node, we return the corresponding pointer and the position.

5.3.2.7.2 Range Lookup

We return all tuples that match, by first finding the first tuple matching, the continue traversal until the last match is found. We can determine the last match easily because the tree's leafs are lexicographically sorted. Since the leafs are linked as linked lists, we can simply move along the linked lists, cutting down on processing time.

5.3.2.7.3 Inserting into the index

We first look up the corresponding leaf. Then, if there is space, we simply insert the value. If there is no space in the leaf, we split the leaf into two new leafs and insert the new item on the corresponding leaf.

All that remains to do now is to insert the separator on the parent node. If that node is full, we split the node in two and insert the separator into the parent node and propagate up to the root, if necessary.

5.3.2.7.4 Deleting from the index

We proceed by looking up the corresponding leaf, then we remove the entry from the leaf. If the leaf now is less than half full, we check the neighboring leaf

- If it is more than half full, we balance the two leafs
- If it is half full, we merge both leaves

For both operations, we update the separator in the parent node.

5.3.2.8 Concurrent Access

Indexes are heavily used while they are being maintained, thus creating conflicting, concurrent accesses.

A way to prevent this is using ***lock coupling***, where we lock a page and its parent and move down the tree.

This prevents problems when the page must be split, however, this is not enough, if we have to go further up to apply the changes.

To prevent this, on every node we check if there is space for one more entry. If not, we split the node. This way, the split of a page never leads to propagating changes.

Alternatively, we can try lock coupling, and if we have to do a change that propagates up, we abort, release everything and start again from the top, but now lock the entire path.

5.3.2.9 Bulk Inserts

To create a B+ Tree index, we proceed as follows (the tree is created bottom up (from the leaves up))

1. Sort the data

2. Use the sorted data to fill one block after the other
3. Remember the largest value in each block
4. create the inner nodes by using the largest value in each block as separator
5. iterate upwards to the next level, until there is only one block left (the root)

If the blocks are filled completely, we get a clustered and compact tree. However, since we usually expect that data has to be updated, it is advised to leave some space in the each block to accommodate that without expensive propagating changes.

5.3.2.10 Optimizations

Reverse Index If the indexed attribute is a sequence and new items are constantly produced, we may encounter an issue where values next to each other go into the same block, leading to concurrent updates fighting to insert. Depending on how updates are handled, many copies could be produced.

Reverse indexing solves that issue by reversing the indices, meaning that consecutive indices go to different pages.

Many Keys Depending on attributes, many keys can become expensive to process. There are several possible optimizations:

- Replace separators that correspond to actual keys with shorter separators with the same effect (can be hard)
- Factor common prefix (this makes a lot of sense and is usually easy, too)

Ignoring rules We can also ignore the rules of the B+ trees and e.g. not merge nodes when they do not have enough data, delaying a merge (and instead rebuilding the tree periodically)

Variable Length Keys This is a bit of a problem. We can mitigate it by no longer forcing the number of entries to lay between $k \div 2$ and k, and using smaller values as separators, placing long values in leafs.

Making sure there is space We don't ever fill blocks to the max and make sure there is space on creation.

Depth vs Breadth For slow devices, prefer larger node sizes, thus shallower trees; For fast devices, prefer deeper trees.

5.3.3 Hashing

5.3.3.1 Hashing in Databases

Hashing is a very common operation in many systems, including databases. Many internal data structures, such as buffer caches and lock tables, are implemented as hash tables. Some operators also use hashing to speed things up, such as hash join and group by operators.

It can also be used as an indexing and partitioning strategy.

Hashing is on the other side compared to B+ trees when it comes to the trade-off between storage and compute, by using more compute, it can save on storage space requirements.

5.3.3.2 Hash Tables

There are many hashing functions with strong properties, but in databases, the hash function has to be very cheap, as it is used very often.

Perfect hash functions do exist, but they need a hash table that is as big as the cardinality of the attributes, so for 4 byte keys, it needs a 4 GB table. Thus, to save on storage, we use a hash table that is smaller than that. If we were to choose it too small, there would be too many collisions, if it is chosen too large, then we waste lots of space.

Collisions may also occur when the index key is not `UNIQUE`, and thus it could be that multiple tuples have the same key.

That may lead you to think, why not grow it if we have a collision. That is a valid idea, but growing hash tables is expensive, thus should be avoided.

Another shortcoming of hash indexes is that it only supports ***point queries***, i.e. it only works for the primary key, but barely anything else.

5.3.3.3 Collisions

A way to handle collisions safely is to use ***chaining***. When a collision occurs, we add another entry in a linked list. If these lists are reasonably short, then it is reasonably efficient. We can also already reserve space for the linked lists to speed things up, if we expect (many) collisions to occur.

Another option is to use ***open addressing***. Here, when a collision occurs, we look for an empty slot in the hash table using some rule. Here, two ways of achieving this are covered in the course:

- ***Linear probing***, where we go to the next slot(s) and place the attribute there
- ***Cuckoo hashing***, where we use several hash functions and move to the next one, if we have a collision with the first, and so on

5.3.3.4 Extensible Hashing

When the hash table is full, in a basic system, we would need to create a new, larger hash table with more buckets (typically double the number) and then rehash all existing items. This of course is very expensive both computationally and in terms of memory accesses, as we do lots of random accesses, which leads to cache misses.

This is where extensible comes in. It is based on sharing buckets and unsharing them when needed. These hash table buckets are mapped into blocks and initially several buckets share the same block

Then, when a bucket gets full, we split the bucket and move entries as needed into a new bucket.

In ***local sharing***, the size of a table can be doubled without immediately adding more space, allowing for more splitting. We simply increase the number of buckets and the degree of sharing and split the buckets as they become full.

Again, if we initially provide more space than needed, we can allow for the first growth to happen without disruption.

We want (ideally) to have two page lookups to access an item, so we typically have a lookup in the bucket directory and the data block and they both can grow independently.

5.3.3.5 Linear hashing

Here, a split pointer is used to indicate which bucket will be split in case of overflow. When it occurs, we chain the block that overflows, split the bucket indicated by the pointer and move the pointer.

This allows for a gradual increase in the size of the table and the idea is to gradually redistribute the data. We always split on the pointer, even if the overflow is elsewhere. The pointer will eventually reach a bucket with chains and when the bucket is split, the data is reorganized.

This has the advantage that the directory (i.e. the list of buckets) grows page by page, instead of doubling.

When all buckets have been split, start fresh.

5.3.3.6 In Postgres

PostgreSQL uses a variation of linear (overflow) hashing and extensible hashing (with doubling). Buckets are initially allocated in groups with powers of two. Overflows are captured in overflow pages (like in linear hashing). When a split occurs, we do not allocate all the pages, but a fraction of them.

These ideas can be combined in many different ways.

5.3.3.7 When to use

For memory structures:

- For shared resources (System Global Area)
- Database Buffer Cache for I/O
- Redo log buffer for recovery
- For large memory pools

Then of course, it can be used for a hash join, for the `SELECT DISTINCT` function and for aggregation (`GROUP BY`).

5.3.4 Bitmaps

Bitmap indexes are used for attributes with low cardinality (low number of distinct values), attributes that are retrieved by category and queries with low selectivity.

In addition, they are used to perform aggregation functions, such as `SUM` and to perform selection and operators over column stores.

Bitmap indexes are an alternative to B-trees and very space efficient.

Bitmaps work as follows:

- List all distinct values of an attribute
- For every distinct value (and for NULLs), create an array with 1 bit entries (i.e. 0 or 1)

- In the array corresponding to a given value of the attribute, set the positions in the array to one that correspond to tuples that have that value in that attribute.
- Now, each array indicates the tuples that have the value corresponding to that array.

5.3.4.1 Bitmaps as indexes

While a B-tree or hash index stores the row ID aor tuple ID, a bitmap index only stores a single bit at a given position, which is indicating the of the tuple from the beginning of the table.

This means that to map a 1 in the beatmap to a tuple address requires applying a function that calcualtes the address. One way to achieve this is to layout the bitmap in page ID order. Then to map a bit position to a record id, we fix the number of records per page, lay the pages out such that the page numbers are sequential and increasing and then construct the row ID, page ID and slot number.

As a result, $\texttt{pageID} = \texttt{bit-pos} \div \#\texttt{records per page}$ and $\texttt{slot-number} = \texttt{bit-pos}\%\#\texttt{records per page}$

We can also compress bitmaps, in Postgres, this is done using run-length encoding.

5.3.4.2 Using bitmaps

Bitmaps are used in database systems to solve simple predicate selections, such as

```
SELECT NAME FROM STUDENTS WHERE Students.Department = D-INFK
```

or for multiple operations, we can perform operations directly on the bitmaps

```
SELECT NAME FROM STUDENTS WHERE Students.Department = D-INFK and Students.Degree = Bachelor
```

In addition, they are usually *very* sparse and long lists with just a few 1s compress very well, using run length encoding.

They are also often used for large tables in analytics as it is faster than other indexes for low selectivity queries and they are especially useful for expensive comparisons, such as for strings.

They can also be used to implement operators:

- `NOT IN`: get a bitmap with the results of the inner query, match it against a bitmap for the outer query, then retrieve the tuples
- `COUNT`: Count how many 1s appear in the bitmap
- `JOIN`: Match (`AND` operation) the bitmaps of the values of the two attributes
- `RANGE` predicates: Combine the bitmaps (`OR` operations) of all the values in the range
- ...

5.3.5 Bloom Filters

A bloom filter is an index that says whether a tuple with a given value is ***NOT*** in the original table. It acts as a filter, eliminating unnecessary accesses to the data. However, it can have false positives, i.e. it may state that a given tuple is there, but it is not in fact. Most importantly, it doesn't have false negatives, which is good, because this means we don't accidentally miss any tuples.

A bloom filter works by creating an array of size m, with values hashed using k hash functions. Each hash point points to a position in the array that is set to 1. Then, to check if an item is in the data, we check all the k hash positions for the value and if they all are all 1, then the item is in the data and if one (or more) is 0, then the element *definitely* isn't in the data.

The bloom filter's error rate depends on the number of hashes (k), the size of the array (m) and the number of items being hashed (n)

5.3.6 Zone maps

There are some systems that do not use indexes, or very sparingly. Instead, they keep metadata about the blocks and store them in memory and use it to check if the data in the block is needed.

This is common in column-store engines such as DuckDB and Snowflake. This metadata can be as simple as min/max for each column, or complete stats, bitmaps, precomputed aggregates, or even bloom filters.

5.3.7 AR Trees

5.3.8 R Trees

5.3.9 Other Optimizations

6 Query Processing

6.1 Sorting

Sorting is crucial in DBMS due to the possibility that the user requests sorted data using an `ORDER BY` clause. In addtion, it can help speed up some operators.

Something like Quicksort is fast, but creates many random accesses, which slows things down. Thus, sorting data as it is loaded into memory is beneficial, for which typically (a variant of) merge sort is used.

6.1.1 Merge Sort

When we sort each page on load into memory, we then have to merge all pages together, or more precisely, combine elements into pages in correct order. For that, we keep pointers to each pair of frames, then we first take first element of either the first or second frame, depending on which one comes first in the order, and copy it into the empty frame. We apply the same again to fill up the empty frame (to a threshold or fully). When the frame is full, we create a second one and link it.

We do this for all pairs, then apply the same procedure to each sorted frame group, repeating this until we have a unified, sorted set of frames.

If there is ever just one frame (group) left, we add an empty one implicitly and proceed normally.

The cost for this is for the 1-page sorted runs (i.e. sorting the original pages) is a single pass (read + write pass).

The total cost is given by $2 \cdot M \cdot N$ I/O operations, with M the number of passes / phases (i.e. how many merge steps there are) and N being the number of frames.

For Two-Way Merge Sort, the number of passes, M is given by $M = \lceil \log_2(N) \rceil + 1$, so the total cost to sort is $2N(\lceil \log_2 N \rceil + 1)$

The primary goal for sorting is of course efficiency. Primarily we want to reduce the amount of extra RAM used, the disk I/O (as that is slow) and trying to reduce random accesses in favour of sequential accesses. And of course, reducing CPU cost is still a concern.

We can increase the number of frames to B for the sorted runs, and then perform a $B-1$-way merge. In that case, the number of operations are given by $2N(1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil)$

6.1.2 Replacement Sort / Heap Sort

It has the advantage that it can produce longer runs with the same amount of memory, on average $2B$ (instead of B) with B frames of memory. It is used as the sorting algorithm in pass 0 above.

It works as follows:

Algorithm 4 Replacement Sort

```
1 procedure ReplacementSort(R)
2     Read B − 1 pages of relation R, store in a heap (priority queue)
3     for every page do
4         if Top of heap is smaller than the end of the sorted run then
5             Continue with next iteration
6         else if No element is left in memory that is larger than last element of current sorted run then
7             create new run
8         else
9             load page, remove smaller records and place them on the current sorted run, until a frame is freed.
```

Informally, it is searching for an always larger value once it has committed to a certain value. It picks the first value to be the lowest value in the current frames.

A new run is started if all values in the loaded frames are defered or used already.

This means that after the sort run is complete, a merge is still needed.

6.1.2.1 Performance

Best case: $\Omega(N)$ elements in a run (thus a single run, data is already sorted)

Worst case: $\mathcal{O}(B-1)$ elements in a run (reversed data)

Average case: $\Theta(2B)$ elements in a run. In that case, the cost of sorting is $2N(1 + \lceil \log_{B-1} \lceil N/2B \rceil \rceil)$

Thus, Quicksort is fatster, but longer runs often mean fewer passes, saving time.

6.1.3 Sorting with B+ Trees

If a table has a B+ Index on the sorted column, we could retrieve the records in order by traversing the leaf pages. In case the B+ tree is clustered, this is a good idea, if it is not, then it could be very bad (due to random accesses).

The operating principle is very simple, abusing the linked lists in the leafs of the B+ tree. If the data is not clustered, we have one I/O per record, which is bad.

6.2 Selection

Two terms important here are *logical selection*, which describes ***what*** we want to select and *physical selection*, which describes ***how*** the algorithm or procedure works that actually retrieves, or filters, the data.

The options include an *file scan*, where we scan the entire file and thus the I/O cost is the number of pages in each relation. Alternatively, we can use *index scan*, where we use an index to retrieve the matching rows. The cost then of course depends on the index used and if said index can even be used to generate the resulsts needed. We will cover that in more detail now.

6.2.1 Index Scan

- ***Hash Index***: $\mathcal{O}(1)$, we read the bucket and possibly the overflow buckets. Can only be used for equality predicates
- ***B+ Tree Index***: $\mathcal{O}(\log_F(N) + X)$, with F fanout, N the number of leaf nodes and X the ratio of number of selected tuples and tuples per page. X can be up to 1 per selected tuple with an unclustered index. Optimization: we could sort the RIDs.
- ***Bitmap Index***: $\mathcal{O}(\text{size of bitmap index}) + X$, X depends on clustering again

The I/O cost for B+ Tree Index Scan is `tree height` + `# leaf pages` + `# file pages`

Example 6.2.1 Computation example Given a relation R with $N =$ one million records. There are 100 records on a page and we have a B+ Tree with the data entries $< k, rid >$ and it has a capacity of 500 data entries on each leaf. It also has 3 internal node levels and a fill factor of 0.67.

Then the cost is computed as follows:

- 3 internal nodes to parse, $\lceil \log_F(N) \rceil$
- Number of result records $= 1,000,000 * 1\% = 10,000$ (this is the selectivity)
- Number of leaf pages pointing to the results records $= 10,000/(500 \cdot 0.67) = 30$
- Number of pages in the heap file that hold the result records $= 10,000/100 = 30$

Then, the total cost is $3 + 30 + 100 = 133$

6.2.2 Index Matching

The predicates in the queries can contain comparison operators, as well as `AND` and `OR`, allowing conjuncts and disjuncts.

Index Matching is the process of choosing the proper index for the task. For conjuncts only, we can use:

- ***Hash Indexes***. Constraints: Only equality operations and predicate has to contain all index attributes
- ***B+-Tree***. Constraints: The attributes in the predicates need to be prefixes of the search key

Here, the indexes need to match ***every*** predicate in the disjunction. For instance, if we have a hash index on A and a B+ tree on B, and the query is `A = 7 OR B > 5`. In that case, we can either file scan or use both indexes (we fetch the RIDs and take the union)

However, index matching is a hard problem.

6.3 Projection

6.3.1 Duplicate Elimination

This is done using either hashing or sorting. The hashing approach hashes an element and checks in the existing map if this element exists already. If not, it is inserted, else it is skipped. Finally the map is returned, or transformed and returned.

For large datasets, we may want to create a hash table of size B, then run multiple dedupe passes, on each subset where duplicates could be, then return the union'd map.

A sort-based approach is also very easy to understand: We sort the records and then discard, on a run through the sorted records, the ones that are duplicates. However, this tends to be more expensive than the hash-based approach for low

amounts of data, but for larger amounts, very similar. The sort-based approach also has the benefit of producing sorted data, which may be useful for future operators.

6.3.2 Set Operations

6.3.2.1 Union / Union All

$R \cup S$ is the operation to perform. We can again use a sort-based or hash-based approach:

- ***Sort-based***: We first sort both relations on all attributes and merge the sorted relations, while eliminating duplicates (except if `UNION ALL`)
- ***Hash-based***: We first hash-partition both R and S. We then build an in-memory hash table for each partition R_p of R and probe with tuples in corresponding S_p of S and add to the output, if not duplicate. (Again only if not `UNION ALL`)

6.3.2.2 Difference

$R - S$ is the operation to perform. We can again use a sort-based or hash-based approach:

- ***Sort-based***: We first sort both relations on all attributes and merge the sorted relations, while eliminating tuples from R that exist in S
- ***Hash-based***: We first hash-partition both R and S. We then build an in-memory hash table for each partition R_p of R and probe with tuples in corresponding S_p of S and add to the output, if it does not exist in the hash table.

6.4 Aggregation

And again, we have the option to do this via sorting or hashing:

- ***Sorting***: Sort on the attribute. Then scan the sorted tuples, computing running aggregate (such as max/min, average, etc). When a new group is encountered, then we output the aggregate. Alternatively, we can already compute the aggregates on the last step of sorting (the merge) to save some time. The limiting factor then becomes the sorting.
- ***Hashing***: We hash the attribute and now each hash table entry is a group of all records with this value of the attribute. For each one of these groups, we compute the aggregate. If the table is too large for memory, we use a two-step approach, as before

6.5 Join

6.5.1 Simple Join

Algorithm 5 SimpleJoin Algorithm

```
1 procedure SimpleJoin(R, S)
2     for each tuple t_R ∈ R do
3         for each tuple t_S ∈ S do
4             if pred(t_R, t_S) then
5                 add{t_R, t_S} to the join output
```

Here, the max output size is $T_R \cdot T_S$, where T_X is the number of tuples for realtion X.

6.5.2 Nested Loops Join (NLJ)

Algorithm 6 Nested Loops Join Algorithm

```
1 procedure NestedLoopsJoin(R, S)
2     for each page p_R ∈ R do
3         for each page p_S ∈ S do
4             Join the tuples on page p_R with the tuples on page p_S (evaluate the predicate for all pairs)
5             Add matching tuples to the join output
```

The cost here is $P_R + P_R \cdot P_S$ I/Os, where P_X is the number of pages in the relation X. Furthermore, the DBMS should put the smaller relation on the outer loop to improve performance.

6.5.3 Index Nested Loops Join (INLJ)

Algorithm 7 Nested Loops Join Algorithm

```
1 procedure IndexNestedLoopsJoin(R, S)
2     for each page p_R ∈ R do                          ▷ Outer loop
3         for each tuple t_r ∈ p_R do
4             Probe the index on the join attribute of S   ▷ Inner loop
5             Add matching tuples to the join output
```

The cost here is $P_R + T_R \cdot C(I_S)$ I/Os, with P_X the number of pages in the relation X, T_X the number of tuples in said relation and $C(I_X)$ is the cost to probe the index I_X of relation X and depends on the clustering of data and type of index.

This operation needs three buffer frames. 1 for R, 1 for output and 1 to traverse I_S

6.5.4 Block Nested Loops Join (BNLJ)

We have B buffer frames, of which one is reserved for the pages of S and one is reserved for the output, thus the $B-2$ blocks. Each of these blocks houses $P_R/(B-2))$ pages of R, reducing search costs by only scanning S once for every block.

The cost here is $P_R + (P_R/(B-2) \cdot P_S$ I/Os. If however, R fits into memory, we have cost $P_R + P_S$ I/Os. We can reduce that further by building as second step (right after the first for loop) a in-memory hash table for the tuples in the R pages block. This reduces the cost of comparisons, however, we need $(B-2) \cdot f$ memory now.

6.5.5 Block Index Nested Loops Join (BINLJ)

The cost here is $P_R + T_R \cdot C(I_S)$ I/Os. We sort the tuples in each buffer frame on the predicate key, which leads to tuples in R with the same key only requiring a single index search. In addition, tuples with "similar" values in R will likely match with keys on the same pages of I_S, which will already be in the buffer pool.

In practice, the difference between BNLJ and NJS is fairly stark.

Example 6.5.1 $P_R = 500$, $P_S = 1000$, 100 tuples per page and we have $B = 12$ frames. Then:

- `Cost(NLJ)` $= P_R + P_R \cdot P_S = 500 + 500 \cdot 1000 = 500,500$IOs
- `Cost(BNLJ)` $= P_R + (P_R/(B-2)) \cdot P_S = 500 + (500/10) * 1000 = 50,500$IOs

We only need very little more memory for BNLJ, with a big improvement in performance.

Algorithm 8 Block Nested Loops Join Algorithm

```
1 procedure BlockNestedLoopsJoin(R, S)
2     for all B − 2 blocks of pages of R do
3         for each page p_S ∈ S do
4             Join the tuples on page p_S with the tuples in the block
5             Add matching tuples to the join output
```

Algorithm 9 Block Index Nested Loops Join Algorithm

```
1 procedure BlockIndexNestedLoopsJoin(R, S)
2     for all B − 2 blocks of pages of R do
3         sort the tuples in this block.
4         for each tuple t_R in B − 2 block of R do
5             Probe the index I_S on the join attribute of S
6             Add matching tuples to the join output
```

6.5.6 (Simple) Sort Merge Join (SMJ)

We first sort R and S on the join attributes with external merge sort (see Section 6.1). We then read the sorted files and merge. This is of course not great for performance, but if the relations are already sorted, we can skip the first step.

The cost (if there are ***no duplicates***) is $\texttt{Sort}(S) + \texttt{Sort}(R) + P_R + P_S$ I/Os.

If there are duplicates (i.e. the keys are not unique, say we are joining on a column that doesn't have the UNIQUE constraint) then we need to move the pointer for the join back on the second relation to make sure we don't miss any tuples.

The cost now of course isn't linear anymore and worst case quadratic, $\mathcal{O}\left(\texttt{Sort}(S) + \texttt{Sort}(R) + P_R \cdot P_S\right)$.

Compared to BNLJ, its performance doesn't depend on the amount of memory available, but tends to be a tad slower than BNLJ, if BNLJ is provided with ample memory.

6.5.7 Optimized Sort Merge Join ((O)SMJ)

In this section, we are exploring how we can optimize sort and merge with extra memory.

Sorting can be optimized using replacement sort, the output is $\frac{P_R+P_S}{2B}$ sorted runs of length $2B$.

That of course means that the logical optimization is to do a K-way merge to merge the sorted runs and join the relations at the same time. If we have B frames, so we can do a $B-1$ way merge, otherwise, we merge runs from each relation separately to reduce the number of runs until we have less than B sorted runs, then we perform the join.

The cost drops to $3 \cdot (P_R + P_S)$ I/Os, but we need memory such that $B^2 \geq \max\{P_R, P_S\}$

6.5.8 Hash Join (HJ)

This works by using a hash function that maps values into $B = P_R \cdot f$ buckets, with f being the Minister of Magic (up until, and including book 5) factor, also known as the fudge factor, which is additional hash table information stored alongside the tuples.

We first partition the table R into these buckets. Then, we probe the hash table for all tuples of S.

Ideally, we would want each bucket to contain exactly the same number of pages ($N/(B-1)$ pages), but due to to hash collisions and duplicate keys, this is not likely to happen. To take care of this imbalance, we do another pass with a different hash function.

We should *NOT* use hash join if the hash table doesn't fit in memory because then it will become VERY inefficient.

6.5.9 Partitioned Hash Join (PHJ)

If the table doesn't fit into memory, then we can't use a normal hash join because that would be terribly inefficient.

Instead, we partition both tables into buckets and join the corresponding partitions.

Algorithm 10 Partitioned Hash Join

```
1 procedure PartitionedHashJoin(R, S)
2     Partition R into RP partitions, using hash function h on the join key
3     Partition S into SP partitions, using hash function h on the join key
4     Join each partition RP with the corresponding SP partition using BNJL or building a hash table SP or RP in
      memory. This hash function should be different from h.
```

The benefit of this approach is that in the third step, we are only operating on small numbers of records each, which can be done efficiently in memory.

What we ideally do in the above algorithm is to do *recursive partitioning*, i.e. as already *somewhat* described, we partition each partition again, if it has more than one page. This further reduces access times.

The cost for this is $3 \cdot (P_S + P_R)$, thus both OSMJ and PHJ are very similar in performance.

6.5.10 Conclusion

We can use all algorithms for equality joins (equi-joins), but if we have equalities over multiple attributes, then:

- PNL, BNL: work the same
- INL, BINL: build indexes on one, or all attributes of the equality conditions
- SMJ, HJ: Sort or hash on the combination of the attributes

For inequality joins:

- PNL, BNL: work the same
- INL, BINL: need clustered B+ tree index
- SMJ, HJ: don't work at all

7 Vector Search

Vector search is for example used in Semantic Search and of course search engines. It is also used to find relevant data to augment an LLM prompt (Deep Research or whatever that type of feature is called).

7.1 Trees/Clustering

Since nearest neighbour search (NNS) is not viable for large scale datasets (which they typically are), we use clustering algorithms such as K-Means to find K cluster centroids. We then assign to each of the centroids the vectors that belong to this cluster

Then, for search, we can check which cluster(s) the query vector is closest to. Due to the low number of centroids, this is comparatively cheap to do. We can then search all the vectors in the corresponding clusters, which is much cheaper due to the much lower number of vectors to search. Still, Vector Search is a very expensive operation due to the large number of vectors to be searched, even if the search itself is $\mathcal{O}(n)$, with n being the number of vectors to search through.

On insert, we search the the closest centroid and simply assign the vector to that cluster.

7.2 Hierarchical Navigable Small World Index (HNSW Index)

The name used in the lecture was HSNW (Hierarchical Small Navigable World) Index.

This is a layered graph concept, at insert, first all nodes are inserted into layer 0 and each is connected to $M \approx 3-4$ neighbours. Then, the edge count is $\mathcal{O}(N \cdot M)$. With $N = 100$ million and $M = 16$, we have $1.6B$ edges, with a typical memory footprint of $4-8$GB for $100M$ 32-bit Edge IDs. Then, some nodes (typically about $N_{L_k} = N/M^k$ to layer k) are promoted to layer 1, connected to M nearest neighbours in $L1$ only. This process is repeated a few times until the number of nodes in the layer is low enough. This number depends on a number of factors, primarily how coarse grained we want the initial search to be.

Then, after building, we prune edges, in a process called RNG pruning. Here we prune an edge (u, v) if there exist edges (u, w) and (w, v), such that $d(u, w) < d(u, v)$ and $d(w, v) < d(u, v)$. Important is that both conditions have to be met, as the goal is to remove the longest redundant edge from the graph. This is done using a greedy algorithm.

7.2.1 Search

We note that greedy search always makes progress in this index because the graph stays navigable.

HNSW index search works by starting at a fixed entry, then doing *one* greedy hop only. The hop goes towards the neighbour on this layer that is closer to the query. This is often done using Beam Search (see below) to find the K closest vectors.

After some number of hops, there are no more neighbours in layer LN that are closer to check, so descend one layer and repeat. Traversal of the layer costs $\mathcal{O}(M_L)$, with M_L the number of edges on layer L. Higher up layers have fewer edges, thus making traversal cheap.

The parameter `ef_search` controls the recall vs latency. Set to 1 it is approximately greedy, for `ef_search` $\gg 1$ means more effort, i.e. more vectors visited.

7.2.1.1 Beam Search

This is another greedy search algorithm, which chooses the next node to visit from a beam of size N. The goal is to find the closest node of the query, with d the distance to the query.

The procedure is simple:

Algorithm 11 Beam Search

```
1 procedure BeamSearch(G, start_node)
2     Beam ← new beam data structure
3     Beam.add(start_node)
4     while beam in Beam do
5         Check distances of neighbours and add them to Beam
6         Truncate Beam to size max_beam                ▷ This is a tunable parameter
7     return closest
```

7.3 Hash Index

To measure the quality of **ANN** in comparison to **KNN**, we use `recall@k` $= |$`ANN_result` $\cap$ `KNN_result`$| \div k$. This however is not always a good metric, as two vastly different results, one objectively worse can score equally well.

Alternative methods mentioned are `RDE@k` and `TDK@k`, but not elaborated any further.

7.3.1 Performance vs Recall Trade-off

This is a challenging task, primarily focusing on how many vectors we need to visit to achieve a user-specified target recall.

A few explored strategies are:

- ***Uniform autotuning for all queries***: Using learned offline models (e.g. Google CloudSQL VectorAssist)
- ***Different for each query***: A model predicts the search effort parameters for each query or index
- ***Adaptive***: Decide to continue or stop early based on the current search state

7.4 In Databases

The benefit of supporting vector search in DBMS is that this makes it easy to use for the end user and can be optimized for large datasets.

7.5 Filtered Vector Search (FVS)

This allows us to query both structured and unstructured data. In concept, this is **SQL** + Vectors, sometimes plus joins, subquery expressions, multiple vector searches (on both sides of the join). Sometimes, the dataset isn't even a vector + tag stored in main memory.

Indices are typically built on unfiltered data. Thus, we need to think about the proper execution method. Suppose we are to retrieve K rows with filtering:

- ***Pre-Filter***: Filter first, then run `NNS`, and we get the K rows. This is expensive if the filter is not selective.
- ***Post-Filter***: We first run **ANN** search, from which we get K' rows which we then filter to get our K rows. This is a recall and performance challenge, because if $K' \gg K$, then we have a lot of wasted effort. If on the other hand $K' > K$ and thus $\sigma(K') < K$, then we have low recall.

One way around this issue is to apply an *inline filter*, i.e. during **ANN** search, we do vector search, evaluate the stopping condition and apply the filter. This gives us K rows directly and a handy performance uplift to go along with it.

Sounds too good to be true? That's because it is. There are still some issues, namely if filtered out nodes don't participate in the navigation. In that case, the search stops if we reach such a filtered node, but there would be nodes that are closer to the query behind this node, which due to the removal of the filtered out node are now no longer accessible.

This means that we need to have the filtered nodes participate in the navigation, which increases the cost by increasing the number of needed distance computations.

Thus, *filters increase the search effort and difficulty of tuning search parameters*. Selective filters make it harder to find valid nodes.

This is where the `Value-Vector-Correlation` comes in. It captures the relationship between the ***probability of satisfying the filters*** and the ***distance from the query vector***. It doesn't however always correlate positively and the correlation depends on the query. For instance, a query like "Item like a blue T-Shirt and price ¿ $10K" is negatively correlated.

7.5.1 Hash-Tree Index

If we use a Hash-Tree instead of the HNSW index, with filtering the internal node navigation does not change, as the data vectors are only in the leaves, so we simply filter there.

7.5.2 Performance Challenges

The cost of a vector search is given by $f(n_d \cdot c_d, n_f \cdot c_f)$, where n_d is the number of distance computations, n_f is the number of filter evaluations and c_i and c_f are the associated costs.

Depending on the query, the cost very much differs, in that for 20% selectivity, filtering first is faster, while for 50% selectivity, doing the NNS first is faster.

Since the distance computation and associated access costs are relative to the vector size, we could use Principal Component Analysis (PCA) or others to reduce the dimension of the vectors. Alternatively, we can quantize some values, with the drawback of reducing precision. Trees offer more residualization, so the best of both worlds is to score fast with quantized vectors, then re-score with the full precision on just a subset.

A further optimization is to change how much of the set we re-score vs how much of it we score.

Not all dataset and query combinations are equally easy, and filters also change the difficulty.

7.5.3 Filter agnosticism

Filter agnosticism isn't just a single type, it's a spectrum.

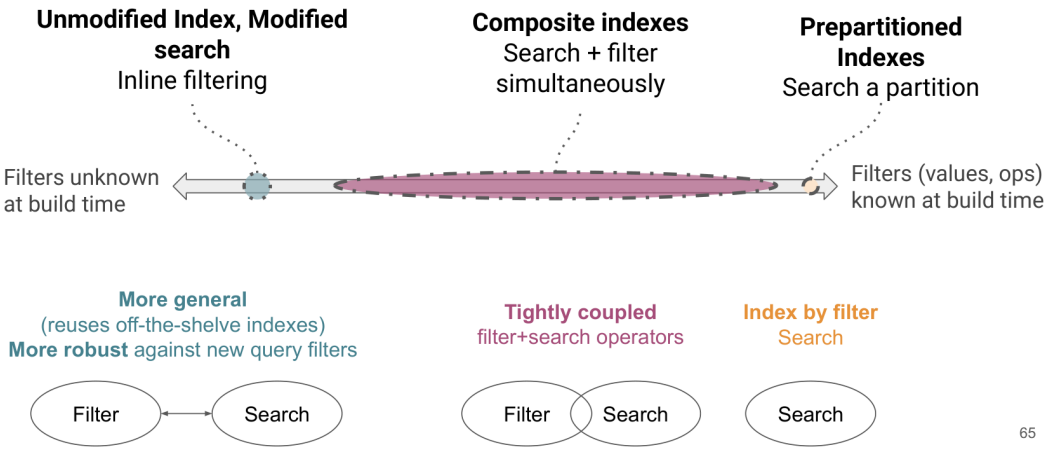


Figure 7.1: Spectrum of filter agnosticism (Figure from lecture slides 18, slide 65)

Ideally, we would build an index per filter, but there are issues:

- Footprint is too high because of duplicated nodes
- Not enough data to build an index on the small partitions

We can emulate this partitioning with efficient footprint using a monolithic graph with pruning.

Definition 7.5.2 **Value-induced neighbourhood** We can build a new index based on attributes and embedding similarity (embedding distance)

Definition 7.5.3 **Predicate Traversal** Alternatively we can reuse unfiltered indexes and use inline filtering to discover filter-passing nodes, then light up the right neighbourhood at search time.

Definition 7.5.4 **Densified Predicate Traversal** We also add edges to alleviate connectivity issues and proceed as with the predicate traversal

8 Data

8.1 Formats

Data formats heavily influence how efficient a database system is. For a 1GB data, JSON is about three times larger than necessary due to field names and text encoding. For the TPC-H lineitem table, the CSV file is 15.95GB, whereas Parquet Snappy and ZSTD are 3.78 and 3.22 Gigabytes respectively, so factor 5 smaller. Furthermore, a raw CSV scanner is about three times slower than a Parquet Scanner.

If we scale that up to Terabytes or more, compute already is dominated by I/O 80-90% of the time, so it really is crucial that we have an efficient format.

8.1.1 Encodings

Typical text encodings are `CSV`, `XML`[3] and `JSON`. Of course, there are also the (in my objective opinion) best formats[4] like `conf` and `yaml`, as well as `TOML`, `INI` and more.

These are good for being human readable, but not if you have terabytes of data. From a practical example, I know that a 1GB JSON file in Python's built in parser takes about 1s to load.

This is why we need (direct) binary encodings for the data, such as the Apache formats.

Examples include Avro (row-oriented format, including a schema describing the fields and types, this is the header of the file), Parquet (Compressed column store), CarbonData (Compressed column store with indexes) and more.

When designing formats, decisions need to be made on the file metadata, format layout, type system, encoding schemes, compression, filters and how to nest the data.

Metadata Ideally the files are self-contained to allow for portability, and for space efficiency reasons (even though it may be beneficial for other reasons), schemas are only stored once per file, in either the header (e.g. for Avro) or in the footer (e.g. Parquet). Other metadata however is more oftne stored once per frame.

The ***format layout*** is the physical organization of the bytes on the disk. We can have row stores, column stores or hybrids:

- N-ary Storage Model (NSM): All attributes stored contiguously in a single page (Row store)
- Decomposition Storage Model (DSM): Store single attributes for all tuples contiguously in a block of data (Column store)
- PAX: Horizontally partition data into row groups, then vertically partition the attributes into column chunks.

For the ***type system***, we choose what kind of types we want to support, be it primitives such as numbers and strings, or more complex types such as arrays. Furthermore, we choose how they are physically represented, such as IEEE-754 encoding for floating point numbers, and logically, if they are complex types based on other types, i.e. how they abstract a more primitive type.

For ***encoding schemes*** include:

- *Dictionary encoding*: We find all unique items of a column and build a dictionary with integer keys and then only store the integers. This is ideal for columns with few distinct values and worse than not doing anything for columns with the UNIQUE constraint set.
- *Run-Length encoding*: If we have many repeating values, called runs, we replace each run with a pair `(count, value)`. This can be done on the data itself, or on the binary representation of said data.
- *Delta Encoding*: Store the first value (as-is), then store the difference between this value and the next. Since these deltas are often small, we can fit them into fewer bits.

For ***Block compression***, we can use typical compression algorithms, such as `LZ4` (which by the way is VERY impressive for compression ratio), `Snappy`, `Zstd` (which `pacman` uses for the packages (`.pkg.tar.zst` files)), or older algorithms such as `gzip` or `xz` (if you remember the whole malware debacle they had, you may not want to use that anymore).

Filters in the files are primarily used to improve traverse performance, by filtering out pages that simply don't need to be searched, using some metadata. For that, statistics are stored in the metadata, such as the min/max, null count, cardinality and more.

[3] you have to be quite a bit of a moron to think that this is the best of the text encodings, but all right
[4] best for configuration files, not (necessarily) in general

8.2 Cloud systems

Cloud systems need to be very reliable, no data is allowed to be lost. In basic versions of so-called ***shared nothing instances***, this is not as much guaranteed as it should be, because all DBs systems have their own disks and they don't replicate or exchange their data.

To improve fault tolerance, make blocks small, like 1MB and replicate each block on a different compute node and ideally on a separate S3 node, which serves as a backup. That's the 3-2 taken care of of the 3-2-1 backup strategy. This should then also be replicated off-site to mitigate the risk of destruction of all nodes from a natural disaster, building collapse, fire or similar.

Modern clouds separate compute and storage entirely. Typically, compute servers have no disk and are booted using iSCSI, or they have a small disk for the base operating system in them.

In systems with disaggregated Compute, Memory and Storage, each layer can scale independently, allowing large memory shuffles to take place and allows for larger numbers of blocks for sorting and joins, which, as we have seen, can massively boost performance.

The Big Query system is one such example.

8.3 Key Value Store

The rise of the internet has shown that there is the need for a system that can more quickly retrieve data than the older implementations using blocks, files and objects. The goal is to be able to access a chunk of data that does not need to be parsed (unlike objects), but can be of various sizes (unlike blocks) and needs no system-supported names (like file systems).

The solution to this problem is called a *Key Value Store* (KVS).

Most **SQL** databases follow the **ACID** concept to maintain consitency, etc. The **ACID** concept gives very strong guarantees, thus the work is done by the database engine as opposed to the application. However, it is expensive, both in throughput and latency and does not scale well.

Key-Value stores get rid of EVERYTHING. They are schema-less, **NoSQL**, no **ACID**, (typically) **BASE** based stores. The most well known examples of **NoSQL** DBs are `Redis` and `MongoDB`, with `Redis` being a true Key-Value Store (however usually referred to as a Key-Value Cache).

8.3.1 Architecture

- Data is stored as a key-value pair. The key is used to locate and retrieve the data. The value is a binary object of arbitrary size
- The data is organized as a distributed hash table. We hash the key to get the location for the value
- KV pairs stored in several places for availability
- Access is **quorum**-based for consistency
- Typically eventual consistency
- The physical storage uses already reserved buckets of different sizes and objects are located in the corresponding buckets.
- This can be run in main memory for very fast access, with larger blobs stored in storage.

8.3.2 Advantages and Disadvantages

Advantages

- Very fast lookups (due to hashing)
- Easy to scale to many machines (simply add more copies / machines). This is great for the cloud's elasticity
- Can be implemented in main memory of machines (giving even faster lookups)
- Useful in many applications (looking up user profiles, retrieving user's web pages and info)
- Scalable and easy to get going (no schema needed)

Disadvantages

- No easy support for queries beyond simple key requests
- this means that there are no joins, no ranges, no queries on attributes, etc (because there are none)
- Depending on implementation, data can be inconsistent
- Maintaining consistency is up to the application
- Used in large scale deployments (worldwide), so keeping data in sync can be challenging

As mentioned above, one major drawback of **NoSQL** DBs is that they move the complexity to the application. This means that some operations are very costly to implement (range queries, joins, etc), as they require reading all data from the DB.

8.3.3 Uses of KVS

As a result of the above disadvantages, KVS are typically used as a cache or specialized system and not as a replacement for **SQL** databases.

They are often used as a main memory cache to avoid having to access the cloud storage, resulting in higher throughput and lower latency.

This is especially useful for commonly run queries, such as loading all videos of a commonly accessed YouTube Channel. In that case especially it is easy to invalidate the cache line and updating it, as updates are very rare and they can trigger a cache invalidation for that channel.

8.3.4 Distributed KVS

How do we distribute a KVS across multiple systems? A simple strategy would be to assign data to machines using modulo n for n machines on the hash. In theory that is not a bad idea, but what if we remove one machine? Or if we add one? Then we have to move ALL of the data. If one machine fails, in the worst case, all data could possibly need to be re-hashed, possibly moving ALL of the data to different machines.

A better solution is to use horizontal partitioning of the data using hashing. Each machine deals with the data points in the clockwise direction before the next machine. In other words, each machine deals with the data in a certain range of hashes, the next machine deals with the subsequent ones, etc.

Thus, on fail, or exit in any other way, only the machine after the failed / exited machine needs to modify the data. When adding nodes, the next machine hands some of the responsibilities over to the newly inserted machine.

To account for possible failure of machines, we need to keep multiple copies over multiple machines (ahead in counter-clockwise orientation) to prevent data loss.

8.4 Cloud-Native Databases

Many of the traditional database engines may run into overhead due to heavy I/O traffic, especially when mirroring.

As a result, many companies have improved implementations of SQL databases to make them suitable to cloud infrastructure.

Some of the improvements include caching. Micro-partitions in the Snowflake system is a system to facilitate query processing. Each one of these partitions has sizes between 50 and 500 megabytes, and has metadata describing what is inside, which can also be read without reading the entire partition.

Furthermore, Amazon S3, the storage platform, does not support in-place updates, files are replaced in their entirety. This means that old data can easily be recovered.

8.5 The Future

"The IT industry is, above all, an industry". This is very much true and trends and developments are driven by demand and market. For hardware, the investment is very high for new developments and only pay off at large sale volumes. Hardware moves slowly, software even slower.

Furthermore, the number of foundries has gone down significantly since 2002 (130nm architecture), where today, there are only three bleeding edge foundries left, of which one has over 50% market share in bleeding edge semiconductor manufacturing, TSMC. Samsung and Intel both have lost out on major deals due to a number of issues (Nvidia Ampere was the last major GPU series manufactured by Samsung).

CPUs are low latency (very fast), but low throughput in terms of parallelism, whereas GPUs are the exact opposite.

Glossary

ACID Atomicitty, Consistency, Isolation, Durability.

ANN Approximate Nearest Neighbour Search.

AVX The Advanced Vector Extensions are an extension to the x86 architecture, to allow CPUs to execute SIMD instruction (commonly called Vector Instructions).

AVX Advanced Vector Extensions, also see **AVX**.

bag A bag, or multiset, is a set, in which, (as is usual with sets) order does not matter, but duplicates are allowed.

BASE Basically Available, Soft State, Eventually Consistent.

DBMS Database Management System.

declarative Describes *what* to achieve, but not how.

imperative Describes how to achieve something.

KNN K-Nearest Neighbour Search.

NoSQL SQL-less database (commonly a Key-Value Store, see Section 8.3).

quorum See this Wikipedia article. TL;DR: It is a synchronization system for distributed computing. .

RA Relational Algebra, see Section 2.2.

record .

schema A set of relations.

SIMD Single Instruction Multiple Data.

SQL Structured Query Language.

subquery Also Nested Query, a separate query that can be used in the `FROM` or `WHERE` clause of another query. See 3.3.2.1.

superkey See Section 4.1.