Autonomous Mobile Robots

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1 Introduction

1.1 Probability

D 1.1 *(Sum rule)* $P(X) = \sum P(X, Y) = \sum P(X \cap Y)$

D 1.2 *(Prod)* $P(X, Y) = P(X|Y)P(Y) = P(Y|X)P(X)$

T 1.3 *(Bayes)* $P(Y_i|X) = \frac{P(X|Y_i)P(Y_i)}{\sum_{j=1}^{n} P(X|Y_j)P(Y_j)}$

D 1.4 *(Cont. Var)* Sums become integrals
e.g. $\sum_X P(X) = 1$ becomes $\int p(x)\ \mathrm{d} = 1$

D 1.5 *(Indep.)* x, y indep. iff $p(x, y) = p(x)p(y)$

D 1.6 *(Cond. Indep.)* iff $p(x, y|z) = p(x|z)p(y|z)$

D 1.7 $E[\boldsymbol{x}] = \int_{-\infty}^{\infty} \boldsymbol{x}p(\boldsymbol{x})\ \mathrm{d}\boldsymbol{x}$, also for $\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x})$

D 1.8 $\mathrm{Cov}[x] = E[\boldsymbol{x}\boldsymbol{x}^\top] - E[\boldsymbol{x}]E[\boldsymbol{x}]^\top = \boldsymbol{\Sigma}$

D 1.9 *(Gauss. Dist.)* $\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ ($\boldsymbol{\mu}$ mean, $\boldsymbol{\Sigma}$ cov.),
PDF: $p(\boldsymbol{x}) = \frac{1}{\sqrt{(2\pi)^k|\boldsymbol{\Sigma}|}} \exp\left(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})\right)$

1.2 Measurement models

$\boldsymbol{z} = \boldsymbol{b}_C + s\boldsymbol{M}_S\boldsymbol{\omega} + \boldsymbol{b} + \boldsymbol{n} + \boldsymbol{o}$: $\boldsymbol{b}_C$ const bias, $\boldsymbol{b}$ time bias, $\boldsymbol{M}$ missal., $\boldsymbol{n} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R})$ noise, ${}_S\omega$ corr. meas., $\boldsymbol{o}$ other infl.

1.3 Trigonometry

2 Locomotion & Kinematics

2.1 Positioning

D 2.1 *(Position Vector)* ${}_{W}\boldsymbol{t}_{B} = {}_{W}\boldsymbol{t}_{WB}$, Original Frame, End point, Target Frame, $\sin = s$, $\cos = c$

D 2.2 *(State vector)* x_R: x, v of rob in W, pos of sensors

D 2.3 *(Rot. Mat.)* $\boldsymbol{R}_z = \begin{bmatrix} c(\psi) & -s(\psi) & 0 \\ s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\boldsymbol{R}_y(\theta) = \begin{bmatrix} c(\psi) & 0 & s(\psi) \\ 0 & 1 & 0 \\ -s(\psi) & 0 & c(\psi) \end{bmatrix}$ $\boldsymbol{R}_x(\varphi) \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\psi) & -s(\psi) \\ 0 & s(\psi) & c(\psi) \end{bmatrix}$

R 2.4 Application: ${}_W\boldsymbol{a} = \boldsymbol{R}_{WB\,B}\boldsymbol{a}$

L 2.5 $\boldsymbol{R}_{BW} = \boldsymbol{R}_{WB}^{-1} = \boldsymbol{R}_{WB}^\top$, $\det(\boldsymbol{R}_{WB}) = 1$ (orth.)

R 2.6 Cols of $\boldsymbol{R}_{WB}$ are basis vec. of Frame $\underset{\rightarrow}{\mathcal{F}}_B$ in $\underset{\rightarrow}{\mathcal{F}}_W$

D 2.7 *(Euler Angles)* Yaw (z), Pitch (y), Roll (x), mult. rotation matrices, e.g. $\boldsymbol{R}_{EB} = \boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_y(\theta) \cdot \boldsymbol{R}_x(\varphi)$, bound..
$[\boldsymbol{n}]^\times = \boldsymbol{n}\boldsymbol{x}^\top$ (matrix from vec + arg $\boldsymbol{x}$)

D 2.8 *(Rot. Vec)* $\boldsymbol{\alpha} = \alpha\boldsymbol{n}$ ($\boldsymbol{n}$ normal)
$\boldsymbol{R}(\alpha, \boldsymbol{n}) = \boldsymbol{I}_3 + \sin(\alpha)[\boldsymbol{n}]^\times + (1 - \cos(\alpha))([\boldsymbol{n}]^\times)^2$

D 2.9 *(Quaternions)* $q = q_w + q_x i + q_y j + q_z k$ with $i^2 = j^2 = k^2 = -1$, ($ij = -ji = k$, same for jk and ki)

D 2.10 *(Transf. M)* $\boldsymbol{T}_{AB} = \begin{bmatrix} \boldsymbol{R}_{AB} & {}_A\boldsymbol{t}_B \\ \boldsymbol{0}_{1\times3} & 1 \end{bmatrix}$

$\boldsymbol{T}_{BA} = \boldsymbol{T}_{AB}^{-1} = \begin{bmatrix} \boldsymbol{R}_{AB}^\top & -\boldsymbol{R}_{AB}^\top {}_A\boldsymbol{t}_B \\ \boldsymbol{0}_{1\times3} & 1 \end{bmatrix}$ $\boldsymbol{T}_{AC} = \boldsymbol{T}_{AB}\boldsymbol{T}_{BC}$

2.2 Forward Kinematics (FK)

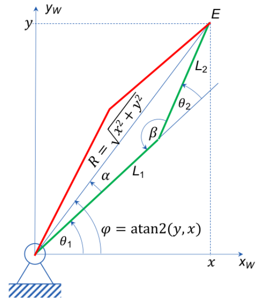
$T_{WB_n}(\theta) = \boldsymbol{T}_{WB_0}\boldsymbol{T}_{B_0B_1}(\theta_1)\cdots\boldsymbol{T}_{B_{n-1}B_n}(\theta_n)$.

For 2R system: ${}_W\boldsymbol{t}_{WE} = \begin{bmatrix} L_1\cos(\theta_1) + L_2\cos(\theta_1 + \theta_2) \\ L_1\sin(\theta_1) + L_2\sin(\theta_1 + \theta_2) \end{bmatrix}$

With workspace (pos) W for $\theta_1, \theta_2 \in [-\pi, \pi]$

2.3 Inverse Kinematics (IK)

Option: Solve Forward Kinematics for angles.
Better: Law of cosine with polar coordinates. Compute angle using cosine rule,
$\theta_1 = \phi \pm \alpha$, $\theta_2 = \pm(\pi - \beta)$
(Positive for Elbow Down, Negative for Elbow Up)
Extension to 6R: 1. Waist: spherical coords (2 sol.)



2. 2 sols from 2R for shoulder + elbow
3. Solve for wrist joints (no influence on pos)

2.4 Temporal Models

Often use cont. time n.-lin. system of ODE $\dot{\boldsymbol{x}} = \boldsymbol{f}_C(\boldsymbol{x}(t), \boldsymbol{u}(t))$, with measurements $\boldsymbol{z}(t) = \boldsymbol{h}(\boldsymbol{x}(t)) + \boldsymbol{v}(t)$. Need linearised (around $\boldsymbol{f}_C(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}}) = 0$, at ***equilibrium***):
$\delta\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_C(\overline{\boldsymbol{x}}, \overline{\boldsymbol{u}}) + \boldsymbol{F}_C\delta\boldsymbol{x}(t) + \boldsymbol{G}_C\delta\boldsymbol{u}(t) + \boldsymbol{L}_C\boldsymbol{w}(t)$
$\delta\boldsymbol{z}(t) = \boldsymbol{H}\delta\boldsymbol{x}(t) + \boldsymbol{v}(t)$. Herein, $\boldsymbol{H}$ is measurements, $\boldsymbol{F}_C$ system, $\boldsymbol{G}$ input gain, $\boldsymbol{w}$ process noise, $\boldsymbol{v}$ measurement noise

2.5 Rigid body & IMU kinematics

2.6 Rigid Body Dynamics

2.7 Wheeled robot Kinematics