

Data Modelling & Databases

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```
1 SELECT d.user_id AS user_id,  
2        d.analytics AS analytics,  
3        d.survey_id AS surveyid  
4 FROM (  
5     ((annotator NATURAL JOIN app_user) users  
6     JOIN survey_users su ON su.users_id = users.user_id) u  
7     JOIN survey s ON u.survey_id = s.id  
8 ) d  
9 WHERE d.analytics <> ''  
10 AND d.analytics LIKE '%x%'  
11 AND d.title <> 'testing'  
12 AND d.further_options <> '';
```

The above query is not what SQL technically is meant for and the data format isn't great.¹

“A curry can be written in many different ways”

- Prof. Dr. Gustavo Alonso, 2026

FS2026, ETHZ

Summary of the Lecture Slides

<https://systems.ethz.ch/education/courses/2026-spring/dmdb.html>

¹However, I simply did not want to come up with a long query only for the title page. This query may or may not be from [here](#).

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1 Introduction

A note on the quote on the title page: He did not *actually* mean “curry”, but of course “query”, but he just pronounced it that way.

This summary assumes some basic knowledge about databases. All words written in **bold** should be clickable and should link you to the corresponding term in the glossary. If I did not forget a term that is, of course.

1.1 IMS

Early databases were not relational and they also required the user of the system to know *how the data was stored*. This of course is not ideal, as any update to the data structure also required an update to the code interacting with it. In addition, manual query optimization has to be done.

Non-relational databases commonly have data redundancy, due to non-hierarchical applications being forced into a tree-based model. This causes issues when updating the data, as one needs to update the data in all places where a copy of the data exists.

1.2 Network Model

In a network model, one **record** is fetched at a time, so it essentially traverses the network. However, we again do not have data independence, thus the application again needs to change when the physical data representation changes. In addition, manual query optimization has to happen again, thus, any change to the physical data representation may cause the need to redo the optimization.

1.3 Relational Model

This is the focus of this course

The relational model has the benefit of having data independence: The application doesn't know how the data is stored and represented. The queries stay the same, even if the data representation changes. In addition, the end user does not have to worry about optimization too much. This, of course, only applies to a limited extent.

2 Relational Model, Logic & Algebra

2.1 Relational Model

In the relational model, relations are sets of tuples (similar to **records**). They are denoted $R(f_1 : D_1, \dots, f_n : D_n)$, or sometimes simply $R(D_1, \dots, D_n)$. An **instance** is a **set** of tuples $I_R \subseteq D_1 \times \dots \times D_n$.

Intuition: You can think of the instance to be the rows in the table.

Important We define relations as **mathematical sets**. This means that they cannot contain duplicates and the order does not matter. Database engines may elect to do this differently, but this course assumes set semantics for the relational model.

2.1.1 Keys

Definition 2.1.1 Candidate Key The **minimal** set of fields that identify each tuple uniquely (non-duplicate attributes (fields) typically). In other words, it is the minimal set such that for its closure F^+ and the relation $\mathcal{R}$ that it belongs to, $\forall y \in F^+ \implies y \in \mathcal{R}$.

Definition 2.1.2 Primary Key A single candidate key, i.e. just a single field. We mark the primary key using underlining in visual **schemas**. Formally, we write $R(\underline{k} : D_k, a : D_a, b : D_b)$ for a given relation, with all valid instances fulfilling

$$I \subseteq D_k \times D_a \times D_b \wedge \forall (k, a, b), (k', a', b') \in I : k = k' \rightarrow (a, b) = (a', b')$$

i.e. if the key is equal, so is the rest of the tuple (thus they are one and the same²).

2.1.2 Queries

We write queries in a relational model using the following set notation (using **SQL** for an actual DB):

$$\{(r, p) \mid r \in \text{Supplies} \wedge p \in \text{Parts} \wedge r.\text{pid} = p.\text{pid} \wedge p.\text{name} = \text{"A"}\}$$

The query language thus processes an entire set at a time and applies set operations over the relations.

² Heidrun is her name

2.2 Relational Algebra

2.2.1 Operators

- **Union** $\cup$: $x \in R_1 \cup R_2 \Leftrightarrow x \in R_1 \vee x \in R_2$ (All tuples from both sets (Only valid for same schemas))
- **Difference** $-$: $x \in R_1 - R_2 \Leftrightarrow x \in R_1 \wedge x \notin R_2$ (Tuples that appear in R_1 , but not in R_2)
- **Intersection** $\cap$: $x \in R_1 \cap R_2 \Leftrightarrow x \in R_1 \wedge x \in R_2$ (Tuples that appear in both sets)
- **Selection** σ : $x \in \sigma_c(R) \Leftrightarrow x \in R \wedge c(x) = \text{true}$, with c a predicate on the passed in set. (Tuples that fulfil the predicate c)
- **Projection** Π : $\Pi_{A_1, \dots, A_n}(R)$ (Keep only a subset of the columns, e.g. for columns `name`, `pid`, $\Pi_{\text{name}}(R)$ returns only the column `name`)
- **Cartesian Product** $\times$: $(x, y) \in R_1 \times R_2 \Leftrightarrow x \in R_1 \wedge y \in R_2$ (Primarily used to express join operations. This however simply joins together the two tables (i.e. similar to expanding terms in maths))
- **Renaming** ρ : $\rho_{B_1, \dots, B_n}(R)$ (Change the name of the attributes of R to B_i)
- **Natural Join** $\bowtie$: $R_1(A, B) \bowtie R_2(B, C) = \Pi_{A, B, C}(\sigma_{R_1.B=R_2.B}(R_1 \times R_2))$. (Join two relations into a table on a column. Natural join joins on columns with same name in both (or all) relations) Edge cases:
 - No shared attributes $R(A, B, C), S(D, E)$: $R \bowtie S = R \times S$
 - All attributes shared $R(A, B, C), S(A, B, C)$: $R \bowtie S = R \cap S$
- **Theta Join** $\bowtie_\theta$: $R_1 \bowtie_\theta R_2 = \sigma_\theta(R_1 \times R_2)$ (Join with custom predicate θ)
- **Equi-Join** $\bowtie_{A=B}$: $R_1 \bowtie_{A=B} R_2 = \sigma_{A=B}(R_1 \times R_2)$ (Join column A with column B)
- **Semi-Join** $\ltimes_C$: $R_1 \ltimes_C R_2 = \Pi_{A_1, \dots, A_n}(R_1 \bowtie_C R_2)$, with $R_1(A_1, \dots, A_n)$ and $R_2(B_1, \dots, B_m)$ (Returns columns only from one side if there is a match in the join)
- **Relational division** $\div$: $R \div S = \Pi_{R-S}R - \Pi_{R-S}((\Pi_{R-S}R) \times S - R)$. In other words, $R \div S = T$, with T being the *largest* relation such that $S \times T \subseteq R$. (Find all rows that do not fulfil a condition)

If we apply set semantics generally, then each operator returns results that adhere to that concept, even Projections!

Semi-Join Reduction A useful trick with semi-joins. It is especially useful in distributed DB, where R and S are on different machines. Suppose we want to Join $R(A, B)$ with $S(B, C)$ on B , then we can do the following $R \ltimes S = (R \ltimes \Pi_B S) \bowtie S$.

The idea is to send the column on which the relations are to be joined from the second system to the first, there execute a semi-join (thus only returning the contents of R that matched with contents of S), then sending only that data to the second machine to finish the full join operation.

Of course, in the real world, users of DB systems don't write relational algebra, as in this case, the order of operations matters for performance, whereas we want database systems to figure this out by themselves.

3 SQL

SQL, or unabbreviated, Structured Query Language, is a **declarative** programming language, used to describe and operate on databases.

This is the point to mention the [xkcd standards comic](#), because, as is the case with almost any standard, people deviate from it and new standards form.

Even though *most* **SQL** databases support the standard SQL operations, many have added a lot of features on top.

SQL is split up into the following parts:

- Data Definition Language, used to create, modify and delete schemas.
- Data Manipulation Language, used to insert, update and delete data
- Query Language, used to retrieve data

3.1 Data Definition Language (DDL)

The DDL is used to describe the schema of relations, in **SQL** called tables. For that, we provide a table name, the columns and their data types.

SQL supports the following data types (plus more):

- | | | |
|--------------|------------|--------------------------------|
| ▪ CHAR(n) | ▪ BIGINT | ▪ BLOB(n) (for large binaries) |
| ▪ VARCHAR(n) | ▪ SMALLINT | ▪ DATETIME |
| ▪ INT | ▪ FLOAT | ▪ MONEY |

3.1.1 Creating Tables

Use the DDL CREATE TABLE keyword to create a table. We can set default values for certain attributes, or can add constraints to them.

Since the primary key must be unique for each entry, it may be useful to configure the primary key attribute with the following constraints PrimaryKeyField INT NOT NULL AUTO_INCREMENT

File: code/sql/ddl/create.sql

```

1 CREATE TABLE TableName (
2     Attribute integer UNIQUE,
3     OtherAttribute varchar(30) NOT NULL, -- Not null
4     NextAttribute character(2) default "AP", -- default value, if unset on insert
5     PRIMARY KEY (Attribute), -- primary key, as in RA
6     CHECK (Attribute > 1)
7 );
```

Note that PostgreSQL doesn't support AUTO_INCREMENT constraints, instead use the SERIAL (or BIGSERIAL) type.

3.1.2 Updating / Altering Tables

Use the DDL ALTER TABLE keyword to update a table's schema. Be aware that if we ADD a column to the schema, unless a default value is provided, the column is set to NULL (see 3.3.4)

File: code/sql/ddl/alter.sql

```

1 ALTER TABLE TableName ADD COLUMN (col integer);
2 ALTER TABLE TableName ADD COLUMN (AnotherColumn integer default "");
3 ALTER TABLE TableName DROP COLUMN AnotherColumn;
```

Instead of a ADD COLUMN or DROP COLUMN, we can also use ADD CONSTRAINT and DROP CONSTRAINT to update constraints

3.1.3 Deleting Tables

File: code/sql/ddl/delete.sql

```

1 DROP TABLE TableName;
```

3.1.4 Constraints

SQL supports a number of constraints that can be put on the different attributes of a schema.

- **NOT NULL**: Disallows this column to be set to NULL, or have a NULL value
- **UNIQUE**: The value of each value in this column must be unique, but it can be NULL. An arbitrary number of columns can have NULL and still be valid
- **PRIMARY KEY**: Sets this column as the primary key. NOT NULL and UNIQUE is set on it implicitly. Contrary to common intuition, it can be set on multiple columns.
- **CHECK c**: Check that the values fulfil a condition. This condition c has the same syntax as for the WHERE clause (see 3.3.1)
- **REFERENCES table**: A Foreign Key, used to refer to another tuple from a different relation. It is typically referencing the primary key of the other table.

For REFERENCES, there are many options for maintenance, which can be set on creating a reference, using ON UPDATE and ON DELETE:

- **Cascade**: Propagate UPDATE and DELETE
- **Restrict**: Prevent deletion of the primary key before the change, causes error
- **No Action**: Prevent modifications after attempting change, causes error
- **Set default / Set null**: Set references to NULL or default value

The default for ON DELETE is NO ACTION.

Example 3.1.4

```
CREATE TABLE tab (id integer REFERENCES OtherTable ON DELETE cascade ON UPDATE cascade)
```

Intuition: For a deletion of a row with NO ACTION set (the default), if there exist any rows of tables with a Foreign key on this row, where the Foreign Key is equal to the to be deleted primary key, an error is thrown. If RESTRICT is set, the error happens a earlier, as the check is performed *before* the deletion is attempted. This has the downside of being possibly a bit slower, if large numbers of unreferenced rows are deleted.

3.2 Data Manipulation Language (DML)

The DML is used to insert, update and delete data from the database.

Below some examples showing the use of the common operations. Note that you can use a WHERE clause from the query language to filter data for both DELETE and UPDATE statements.

File: code/sql/dml.sql

```

1  INSERT INTO TableName (
2      Attribute,
3      OtherAttribute
4  ) VALUES ( 1000, 'Value' );
5
6  DELETE FROM TableName WHERE Attribute > 50;
7
8  UPDATE TableName SET Attribute = Attribute + 1;
9
10 -- Import data from a CSV file (this is postgres specific syntax)
11 COPY TableName FROM '/data.csv' WITH FORMAT csv;

```

3.3 Query Language

The basic structure of a **SQL** statement is `SELECT I FROM T WHERE C`, which corresponds to $\Pi_I(\sigma_C T)$.

We can set $I = *$ if we want to return all columns for a table. To rename, we can set $I = \text{Column as Name, Column2 as Name2}$, etc

Theorem 3.3.1 Every SPJR **RA** expression can be written in `SELECT ... FROM ... WHERE ...` form:

Operation	Notation	SQL
Selection	$\sigma_c(R)$	<code>SELECT * FROM R WHERE c;</code>
Projection	$\Pi_{A_1, \dots, A_n} R$	<code>SELECT A1, ..., An FROM R;</code>
Cartesian Product	$R_1 \times R_2$	<code>SELECT * FROM (R1, R2);</code>
Rename	$\rho_{a,b,c} R$	<code>SELECT A as a, ..., B as c FROM R;</code>
Union	$R_1 \cup R_2$	<code>R1 UNION R2;</code>
Difference	$R_1 - R_2$	<code>R1 EXCEPT R2;</code>
Intersection	$R_1 \cap R_2$	<code>R1 INTERSECT R2;</code>

Since **SQL** implements **bag** and not set semantics, there is a **DISTINCT** keyword, which is used to remove duplicates. It is to be applied in the **SELECT** clause (I above), e.g.

```
SELECT DISTINCT name FROM data_table;
```

For the last three operations in the table above, R_1 and R_2 typically are other **SQL** statements, example:

```
(SELECT name FROM FirstTable) UNION (SELECT name FROM SecondTable)
```

In addition, to apply *bag semantics*, as opposed to *set semantics* append **ALL** to the **UNION** expression.

We can also name the tables, e.g. $T = \text{One } o, \text{ Two } t$, and then access columns from a specific table using $I = o.Col, b.Col$, or the like.

String concatenation works using `CONCAT('string', 'string', 'string', ...)`, or using `'string' || 'string' || ...`. Note that SQL uses single quotes for Strings, and double quotes for renaming columns (using **AS** statements, or in **SELECT**)

Dates work as you'd expect. We can create a new date using `Date('ISO-date-string')`. **DATETIME** combines date and time and **TIME** is just the time. To compute time delta, we can use `DATEDIFF('interval', DateOne, DateTwo)`, where the interval can be: year, quarter, month, dayofyear, day, week, weekday, hour, minute, second, millisecond. To get just the year from an existing date object (or string), use **YEAR**. The result is an integer and thus you can use comparison operations with numbers.

The current date, time, year, etc is provided using `current_date` (etc).

3.3.1 Basic operators

3.3.1.1 Logical Operators

The following **SQL** logical operators are available (may not be exhaustive for all systems, *cop* is a comparison operator):

Operator	Description
P1 AND P2	TRUE if P1 and P2 both are true
P1 OR P2	TRUE if one of P1 or P2, or both are true
C <i>cop</i> ALL query	TRUE if all values x returned in subquery query fulfil C <i>cop</i> x
C <i>cop</i> ANY query	TRUE if one (or more) values x returned in subquery query fulfil C <i>cop</i> x. SOME is logically equivalent
C BETWEEN low AND high	TRUE if numerical, text or date value of C is between low and high. Both ends inclusive, negatable
C IN list	TRUE if value of C is in the provided list. List format: (val, val2). List can also be a subquery , negatable
C LIKE R	TRUE if value of C matches regex-like pattern R. (%) matches any number of chars, _ matches a single, arbitray character)
EXISTS query	TRUE if subquery query returns at least one row

3.3.1.2 Comparison & Arithmetic Operators

Comparison Operators <, <=, =, >=, > are defined as usually, <> is defined as not equal.

Arithmetic Operators +, -, *, /, % are defined as usual.

This also means that we can use **SQL** as a calculator, for example::

```
SELECT a * b
```

We can use the arithmetic operators like this:

```
SELECT Value * 1.1 FROM Data;
```

or in WHERE clauses like this:

```
SELECT Value FROM (Data a, Data b) WHERE a.Value = b.Value - 1;
```

Remark 3.3.2 You may have noticed that in the above query, the same table was used twice. This is referred to as a **Self-Join** and can come in handy when comparing values in a single table, or to build pairs.

We can use comparison operators also with subqueries containing aggregations, if they return a single result, for multiple results, need ANY, ALL, etc (see above)

3.3.2 Join

Specifying two (or more) tables in the FROM clause will perform a cartesian product, e.g.

```
SELECT * FROM (Table1, Table2);
```

returns all rows of the tables, next to each other.

Here, naming the tables in the FROM clause may come in handy, to not write out the entire name of the tables:

```
SELECT * FROM (Table1 t, Table2 s) WHERE t.id = s.id;
```

```
SELECT * FROM (Table1, Table2) WHERE Table1.id = Table2.id;
```

Both of the above statements produce the same result.

3.3.2.1 Nested Queries

To not have the cartesian product executed in the FROM clause, or to get a much more complex starting table, we can use a **subquery**. These are parenthesized normal SQL queries.

We can create an alias for them using an AS name clause after the parenthesis as follows

```
SELECT r.name FROM (SELECT * FROM Table) AS r;
```

or alternatively, using a WITH clause (replace the dummy queries with real queries):

```
WITH CteName AS (SELECT * FROM Table) SELECT name FROM CteName;
```

Note that it is possible to have multiple with statements as follows:

```
WITH with1 AS (SELECT * FROM Table1), with2 AS (SELECT * FROM with1) SELECT name FROM with2
```

WITH clauses are an implementation of **Common Table Expressions** (CTE)

3.3.2.2 Join Operations

If we don't specify the kind of join, an INNER JOIN is executed. The following three queries are equivalent

```
SELECT * FROM Student, Tests WHERE Student.PersNr = Tests.PersNr;
```

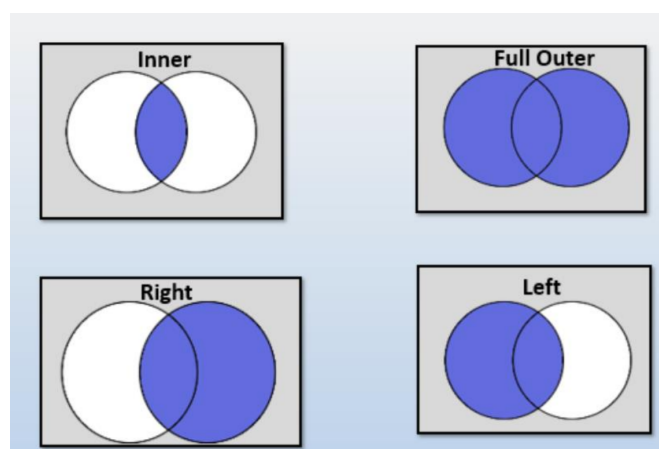
```
SELECT * FROM Student JOIN Tests ON Student.PersNr = Tests.PersNr;
```

```
SELECT * FROM Student JOIN Tests USING (PersNr);
```

For the latter of the three, the parenthesis around the column are important, as omitting them is invalid syntax.

Other types of JOIN are (the latter two are considered to be OUTER JOINS):

- NATURAL JOIN (no ON required), joins on columns with same name
- LEFT/RIGHT JOIN (requires ON), also returns non-matching rows from left or right side, respectively
- FULL JOIN (or FULL OUTER JOIN; requires ON), also returns non-matching rows from both sides



3.3.3 Aggregations, Grouping, Sorting

3.3.3.1 Aggregations

SQL supports (at least) the aggregation functions SUM, MIN, MAX, AVG, COUNT, whose output should be evident.

Please note that you cannot use the aggregation functions and still output another column, as they produce a single value.

For each of the functions, you can optionally specify DISTINCT.

We typically want to specify an AS clause, to make the output nicer to read

3.3.3.2 Grouping

Grouping is used to, well, group together the outputs on a column. Using SQL, this is achieved using the GROUP BY clause. We specify the columns on which the grouping should occur.

IMPORTANT We can only use aggregates and columns appearing in the GROUP BY clause in the SELECT clause.

Using a JOIN function, it is of course then possible to go back retrieve the other columns.

To apply filtering on the columns, the HAVING clause can be used. It applies a comparison to a grouping column or an aggregate as specified in the SELECT clause:

```
SELECT COUNT(name) as cnt FROM Students GROUP BY Department HAVING cnt > 1;
```

3.3.3.3 Sorting

We can sort the output of any query using the ORDER BY column DIRECTION clause, where DIRECTION is to be replaced with ASC or DESC:

```
SELECT name FROM Students ORDER BY name DESC;
```

We can also specify more than one order by condition, separated with a comma.

3.3.3.4 Limit

We can limit the number of columns to return using the LIMIT clause. Example:

```
SELECT * FROM Exams LIMIT 10;
```

3.3.3.5 Example

Below some examples of SQL Queries using grouping, aggregations, sorting and grouping

File: code/sql/aggregations.sql

```

1 SELECT MAX(price) as MAX_PRICE FROM Products; -- Get the most expensive product
2 SELECT AVG(grade) as AVG_GRADE FROM Exams WHERE id = 0; -- Get the average grade for an exam
3
4 -- Aggregation with grouping (Remember, only aggregates and cols in GROUP BY clause can
5 -- appear in SELECT clause!)
6 SELECT id, AVG(grade) as "Average Grade", COUNT(grade) as "Count"
7 FROM ExamResults GROUP BY id; -- Average grade for each exam and number of students taking it
8
9 SELECT id, AVG(grade) as avg, COUNT(grade) as cnt FROM ExamResults
10 GROUP BY id HAVING cnt > 0 ORDER BY avg DESC LIMIT 20;
11
12 -- Get exam details for all exams as filtered before
13 SELECT * FROM (
14     SELECT id, AVG(grade) as avg, COUNT(grade) as cnt FROM ExamResults
15     GROUP BY id HAVING cnt > 0 ORDER BY avg DESC LIMIT 20
16 ) as filtered JOIN Exams e ON filtered.id = e.id;
```

3.3.4 Null

Whenever we have incomplete information, we can set a column to be NULL. If you have a column that has NULL values, when using WHERE clauses, these values are skipped. *However* if you are not filtering on the column, a NULL value is returned.

To check if a column is NULL, you can use the IS keyword:

```
SELECT * FROM Data WHERE name IS NULL
```

- In GROUP BY, all NULLs form a single group.
- COUNT(*), also counts rows with NULL
- COUNT(TheColWithNull) ignores rows with NULL
- Most other aggregations ignore NULL
- If all rows in an aggregations are NULL, so is the result

The OUTER JOIN operators may (and likely will) introduce NULL

3.3.5 Views

Views are a useful way of abstracting a complex query and is the result of a stored query.

We can create views using the following template:

```
CREATE VIEW name AS query;
```

We can then operate on it as if it were a table.

Common applications for views are privacy / access control, because you can limit access to tables only via certain views for every user; Usability is also often much better due to having less complex queries to write - you are just writing them once.

How a view is evaluated This is quite simple in essence, the VIEW's query is inserted into the query using the view. It is then optimized as it normally would be, too. Of course, it is likely that more optimization can and has to be done on views.

3.3.5.1 Updatable Views

Updating Views refers to updating values of the tables in the views. This is not straight forward and thus, many restrictions are placed on what kind of views are considered updatable.

In **SQL**, a view is updatable if and only if it involves **one** base relation and its key, it does **not** involve aggregates, groups or duplicate elimination.

In short, if there is one-to-one mapping between the view and base relation, it is updatable.

4 Theoretical Background

4.1 Functional Dependency

As with relational algebra, we again define a schema to be a relation, e.g. $\mathcal{R}(A : D_A, B : D_B, C : D_C, D : D_D)$, with an instance of it being $R \subseteq D_A \times D_B \times D_C \times D_D$.

Then, we let $\alpha, \beta \subseteq \mathcal{R}$, i.e. they each have a subset of the columns of $\mathcal{R}$.

$\alpha \rightarrow \beta$ (β is a functional dependency of α) if and only if $\forall r, s \in R : r.\alpha = s.\alpha \implies r.\beta = s.\beta$

We write $R \models \alpha \rightarrow \beta$ if R satisfies $\alpha \rightarrow \beta$ semantically, and $R \vdash \alpha \rightarrow \beta$ if the same applies syntactically (i.e. we can apply inference rules over Functional Dependencies).

Intuition: This means that there is a function that maps the values of columns α to the values of columns β .

Definition 4.1.1 $\alpha \subseteq \mathcal{R}$ is a **superkey** if and only if $\alpha \rightarrow \mathcal{R}$.

Intuition: This means that if we know the values of columns α , we know the value of the rest of the columns in $\mathcal{R}$. It is a superset of some candidate keys (see 2.1.1)

Definition 4.1.2 $\alpha \rightarrow \beta$ is minimal if and only if $\forall A \in \alpha : (\alpha - \{A\}) \not\rightarrow \beta$. These are denoted $\alpha \rightarrow \cdot \beta$

4.1.1 Inference

A fundamental result for a given set of Functional Dependencies F is $F \models \alpha \rightarrow \beta \Leftrightarrow F \vdash \alpha \rightarrow \beta$ (assuming $\vdash$ is defined by Armstrong's Axioms)

The goal is to find new FDs that can be implied from a set of FDs F on scheme $\mathcal{R}$. Let $\alpha \rightarrow \beta$ be another FD on $\mathcal{R}$. Then:

- F implies $\alpha \rightarrow \beta$, if every relation instance R of $\mathcal{R}$ that satisfies all FDs in F also satisfies $\alpha \rightarrow \beta$.
- The **closure** F^+ of F is the set of all FDs implied by F .
- Let G be another set of FDs on scheme $\mathcal{R}$. F and G are **equivalent** ($F \equiv G$) if $F \models G$ and $G \models F$

4.1.2 Armstrong's Axioms

- **Reflexivity** $\alpha \subseteq \beta \implies \beta \rightarrow \alpha$, special case: $\mathcal{R} \rightarrow \alpha$ (referred to as *trivial FDs*)
- **Augmentation** $\alpha \rightarrow \beta \implies \alpha\gamma \rightarrow \beta\gamma$, with $\alpha\gamma = \alpha \cup \gamma$
- **Transitivity** $\alpha \rightarrow \beta \wedge \beta \rightarrow \gamma \implies \alpha \rightarrow \gamma$

These three axioms are all complete and sound. All other possible FDs can be implied from these axioms.

- F **derives** $f = \alpha \rightarrow \beta$, denoted $F \vdash f$, if there is a derivation for f using only Armstrong's axioms.
- F **implies** f , denoted $F \models f$, if for every relation instance R of $\mathcal{R}$ that satisfies all FD in F also satisfies f .

Definition 4.1.3 Soundness $F \vdash f \implies F \models f$

Definition 4.1.4 Completeness $F \models f \implies F \vdash f$

Definition 4.1.5 Closure of α with respect to F α^+ is the set of all attributes $y \in \mathcal{R}$ such that $\alpha \rightarrow y$ can be derived from F using Armstrong's axioms:

$$\alpha^+ = \{y \in \mathcal{R} \mid F \vdash \alpha \rightarrow y\} \quad F \vdash \alpha \rightarrow \beta \Leftrightarrow \beta \subseteq \alpha^+$$

4.1.3 Other rules

- **Union of FDs** $\alpha \rightarrow \beta \wedge \alpha \rightarrow \gamma \implies \alpha \rightarrow \beta\gamma$
- **Decomposition** $\alpha \rightarrow \beta\gamma \implies \alpha \rightarrow \beta \wedge \alpha \rightarrow \gamma$
- **Pseudo Transitivity** $\alpha \rightarrow \beta \wedge \beta\gamma \rightarrow \theta \implies \alpha\gamma \rightarrow \theta$

4.1.4 Closure Algorithm

For the definition of the closure, see above.

Finding a closure α^+ using an algorithm³:

Algorithm 1 Finding Closure

```

1 procedure FINDCLOSURE( $F$ )
2    $\alpha^+ \leftarrow \alpha$ 
3   repeat
4      $\alpha_{old}^+ \leftarrow \alpha^+$ 
5     for each FD  $\beta \rightarrow \gamma \in F$  do
6       if  $\beta \subseteq \alpha^+$  then
7          $\alpha^+ \leftarrow \alpha^+ \cup \gamma$ 
8   until  $\alpha^+ = \alpha_{old}^+$ 
9   return  $\alpha^+$ 

```

We can use that to check the following:

- $F \vdash \alpha \rightarrow \gamma$: Calculate α^+ and check $\gamma \in \alpha^+$
- $F \vdash G$: For each $\alpha \rightarrow \gamma \in G$, check $F \vdash \alpha \rightarrow \gamma$
- F equivalent to G : Check $F \vdash G$ and $G \vdash F$
- $K \subseteq \mathcal{R}$ **superkey** for F : Check $F \vdash K \rightarrow \mathcal{R}$:

Minimal Cover

Definition 4.1.6

A minimal cover (or minimum basis) of F is a set of FDs G that has the following properties:

- $G \equiv F$
- All FDs in G have form $X \rightarrow A$, with A being a single attribute
- It is not possible to make G “smaller”, i.e. if deleting a FD, $G - \{X \rightarrow A\} \not\equiv G$, or $G - \{XA \rightarrow B\} + \{X \rightarrow B\} \not\equiv G$, for any FD $XA \rightarrow B \in G$

Computing the minimum basis:

1. G set of FDs obtained from F by decomposing the right hand sides of each FD to single attribute
2. Remove trivial FDs
3. Remove redundant attributes from LHS of FDs in G (by, for each $X \rightarrow Y$ taking each attribute $x \in X$ and, if $X - \{x\} \rightarrow Y$ implies $X \rightarrow Y$, replacing $X \rightarrow Y$ with $X - \{x\} \rightarrow Y$)
4. Remove all redundant FDs

4.1.5 Using Functional Dependencies

Functional dependencies help with decomposing relations into several relations that each contain just one concept, since relations combining several concepts are bad.

4.1.6 Intuition

Adapted from the [Wikipedia Article on this topic](#)

The idea of functional dependencies is to describe which attributes belong to the *determinant set* X and which belong to the *dependent set* Y . The values of the attributes in the *determinant set* determine the values of Y .

Thus, for a relation R , with $X, Y \subseteq R$, with $X \rightarrow Y$:

- When attributes X have known values (let's call them x), then the values of for their corresponding Y attributes can be determined by looking them up in **any** tuple of R containing the values x
- Two tuples sharing the same values of X share the same values of Y .

A FD $X \rightarrow Y$ is *trivial*, if $Y \subseteq X$

³Yes, I did name the algorithm that. Sorry about that, the pun was too good to skip

4.2 Normal Forms

Definition 4.2.1 Normal forms describe the properties of concrete schemas based on functional dependencies in terms of data redundancy and data integrity.

For each of the normal forms, we need to be able to decide:

- whether $\{R, F\}$ satisfies the NF
- given $\{R, F\}$ that satisfies the NF, what are the properties?
- how can we generate new schema R' , such that $\{R', F\}$ satisfies the given normal form

4.2.1 First Normal Form

In the first normal form, only atomic domains are allowed, i.e. keys are not allowed to be arrays.

Intuition: This makes the concept of a key easier to define, or well defined

This can come in handy to solve one to many relationships, but what if many different persons have the same elements there? This can cause some issues when updating.

However, some **SQL** flavours support arrays. However, the semantics and the support varies. Arrays are very tempting to use if we have an unknown and variable number of different data points for a field.

4.2.2 Second Normal Form

For a relation to be in 2NF, every non-key attribute needs to minimally dependent on **every** key, i.e. no attribute depends on part of a key. If relations are not in 2NF, they may experience insert, update and delete anomalies, which can lead to incorrect, redundant or inconsistent updates of the relations.

However, some relations can still suffer from update and / or delete anomalies. One such example is a non-key Functional Dependency.

4.2.3 Third Normal Form

A relation R is in 3NF if and only if for all $\alpha \rightarrow B$, at least one of the following conditions holds:

- $B \in \alpha$ (then $\alpha \rightarrow B$ is a trivial FD)
- B is an attribute of at least one key
- α is a **superkey** of R

Intuition: if $\alpha \rightarrow B$ does not satisfy any of these conditions, α is a concept in its own right, thus, the 3NF tries to get rid of "transitive dependencies" (e.g. $A \rightarrow B, B \rightarrow C$)

The 3NF can still experience update and delete anomalies.

4.2.4 Boyce-Codd Normal Form (BCNF)

R is in BCNF if and only if for all $\alpha \rightarrow B$ at least one of the following conditions holds:

- $B \in \alpha$ (thus, $\alpha \rightarrow B$ is a trivial FD)
- α is a **superkey** of R

Intuition: In each relation, you only store the same information once.

The following FD are okay in both 3NF and BCNF because both left hand sides are candidate keys for the table.

- $\{\text{PersNr}\} \rightarrow \text{Name, Level, Room, City, Street, Canton}$
- $\{\text{Room}\} \rightarrow \text{PersNr}$

4.2.5 BCNF Decomposition Algorithm

Algorithm 2 Decomposition algorithm

```

1 procedure DECOMPOSITION( $\mathcal{R}$ )
2   result  $\leftarrow \{\mathcal{R}\}$ 
3   while  $\exists \mathcal{R}_i$  in result such that  $\mathcal{R}_i$  is not in BCNF do
4     let  $\alpha \rightarrow \beta$  be the evil FD
5      $\mathcal{R}_i^1 \leftarrow \alpha \cup \beta$ 
6      $\mathcal{R}_i^2 \leftarrow \mathcal{R}_i - \beta$ 
7     result  $\leftarrow (\text{result} - \mathcal{R}_i) \cup \{\mathcal{R}_i^1, \mathcal{R}_i^2\}$ 
8   return result

```

In words, this algorithm takes as input a schema $\mathcal{R}$ and outputs a list of schemas $\mathcal{L} = \mathcal{R}_1, \dots, \mathcal{R}_n$, such that $\mathcal{L}$ is a lossless decomposition of $\mathcal{R}$ (i.e. joining them together will perfectly recover the information in $\mathcal{R}$) and each of the $\mathcal{R}_i$ are in BCNF.

Of note is that this does **not** preserve the functional dependencies and some data redundancies will still remain, only the ones caused by functional dependencies.

In addition, the order of picking the evil FD matters, so we use heuristics, which pick the one with the largest right hand side.

4.2.6 3NF Synthesis Algorithm

Algorithm 3 Synthesis algorithm

```

1 procedure SYNTHESIS( $\mathcal{R}$ )
2   Compute the minimal basis  $F_c$  of  $F$ 
3   for all  $\alpha \rightarrow \beta \in F_c$  do
4     create  $R_{\alpha \cup \beta}(\alpha \cup \beta)$ 
5   If none of the above relations contains a superkey, add a relation with a key
6   Eliminate  $R_\alpha$  if there exists  $R'_\alpha$  such that  $\alpha \subseteq \alpha'$ 

```

In words, this algorithm takes as input a schema $\mathcal{R}$ and outputs a list of schemas $\mathcal{L} = \mathcal{R}_1, \dots, \mathcal{R}_n$, such that $\mathcal{L}$ is a lossless decomposition of $\mathcal{R}$ and each of the $\mathcal{R}_i$ are in 3NF.

This algorithm preserves all functional dependencies, however does not remove redundancies because it is not BCNF.

4.2.7 Overview

This overview is adapted from [this Wikipedia article](#). All constraint definitions in smaller font are informal. Note that *prime attributes* are the attributes that make up a candidate key, also known as *key attributes*

Constraint	UNF	1NF	2NF	3NF	BCNF
Unique rows (no duplicates)	✓	✓	✓	✓	✓
Scalar Columns (Columns can't contain relations or composite values)	✗	✓	✓	✓	✓
Every non-prime attribute has a full FD on each candidate key (Attributes depend on the whole of every key)	✗	✗	✓	✓	✓
Every Non-trivial FD begins either with superkey or ends with prime attribute (Attributes depend only on candidate keys)	✗	✗	✗	✓	✓
Every non-trivial FD begins with a superkey	✗	✗	✗	✗	✓

This means that a relation in BCNF is free of redundancies based on functional dependencies.

Note that the above table needs to be read vertically, i.e. for UNF only Unique Rows needs to be satisfied, thus it doesn't mean that if we have Scalar Columns that this isn't allowed in UNF. In fact, it is allowed, as it is a more strict property.

5 Systems

5.1 Query Optimization

A lot of performance can be gained in a database system by executing the query in the correct order. For that, a query optimizer exists, which uses a quite large amount of metadata to speed up query execution.

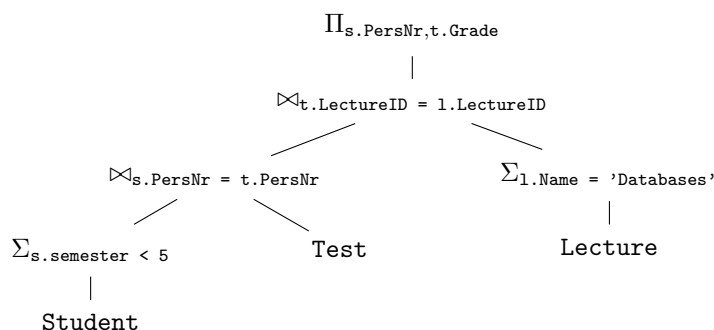
A query optimizer very broadly works as follows:

1. It searches the space of equivalent execution plans. This space tends to be huge, so efficient exploration is needed.
2. Of all the discovered plans, the best is chosen using a cost model, which describes the most efficient plan

5.1.1 Search Space

To create the search space, the optimizer splits a query into a collection of query blocks, where each query block has one SELECT and FROM clause and *at most* one WHERE, GROUP BY and HAVING clause.

Query plans are typically drawn up as trees, as they are typically easier to understand that way than as both SQL query or in relational algebra notation. In addition, the query parser commonly transforms it into a tree-like data structure, so thinking about query plans in this way is a good habit. For the symbols used here, see section 2.2.



5.1.1.1 Rewriting Rules

Given a logical plan as above, there are multiple ways in which we can construct a physical plan. We can use different join orders, decide when selection happens and specific implementations of operators.

The following rules were discussed in the lectures:

R1 Conjunctive selection operators can be deconstructed into a sequence of individual selections:

$$\sigma_{\theta_1 \wedge \theta_2} = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

This is useful to split selections to push them earlier in the execution plan (reduces the number of elements to process subsequently)

R2 Selection operators are commutative:

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

This is useful to allow to first put the one that e.g. has an index.

R3 Only the last in a sequence of projections is needed, others can be omitted

$$\Pi_{t_1}(\Pi_{t_2}(E)) = \Pi_{t_1}(E)$$

This avoids going over the data twice

R4 Selections can be combined with cartesian products and theta joins

$$\sigma_{\theta}(E_1 \times E_2) = E_1 \Join_{\theta} E_2 \quad \sigma_{\theta_1}(E_1 \Join_{\theta_2} E_2) = E_1 \Join_{\theta_1 \wedge \theta_2} E_2$$

This combines the join and selection (thus avoiding having to go over the data twice)

R5 Theta-join operations (and natural joins) are commutative.

$$E_1 \Join_{\theta} E_2 = E_2 \Join_{\theta} E_1$$

This is useful so we can choose as the smaller table as the outer table or as the inner one the one with an index.

R6 Natural join operations are associative, so are theta joins (with restrictions)

$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3) \quad (E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} = E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

This allows for joins to be performed in different orders, allowing us to do the most selective first (fewer rows)

R7 Pushdown selection:

$$\sigma_{\theta}(E_1 \bowtie E_2) = \sigma_{\theta}(E_1) \bowtie E_2$$

This allows us to first remove the data that is not needed before doing the more expensive operations

R8 Projections distribute over theta joins (applies if θ involves only attributes from $L_1 \cup L_2$ of course)

$$\Pi_{L_1 \cup L_2}(E_1 \bowtie_{\theta} E_2) = \Pi_{L_1}(E_1) \bowtie_{\theta} \Pi_{L_2}(E_2)$$

This allows the table width to be reduced before the join to reduce **data movement**

R9 Union and Intersection are commutative

$$E_1 \cup E_2 = E_2 \cup E_1 \quad E_1 \cap E_2 = E_2 \cap E_1$$

This allows us to perform one side before the other, depending on resource constraints

R10 Set union and intersection are associative

$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3) \quad (E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

This allows us to change the order in which operations are done and how the operator tree is executed.

R11 The selection operation distributes over $\cup, \cap$ and $-$

$$\sigma_{\theta}(E_1 - E_2) = \sigma_{\theta}(E_1) - \sigma_{\theta}(E_2) \quad \sigma_{\theta}(E_1 \cup E_2) = \sigma_{\theta}(E_1) \cup \sigma_{\theta}(E_2) \quad \sigma_{\theta}(E_1 \cap E_2) = \sigma_{\theta}(E_1) \cap \sigma_{\theta}(E_2)$$

In addition, we have

$$\sigma_{\theta}(E_1 - E_2) = \sigma_{\theta}(E_1) - E_2 \quad \sigma_{\theta}(E_1 \cap E_2) = \sigma_{\theta}(E_1) \cap E_2 \quad \text{not for } \cup$$

This allows us to remove unneeded tuples before performing the next operation, which may be involving comparisons

R12 The projection operation distributes over union

$$\Pi_L(E_1 \cup E_2) = \Pi_L(E_1) \cup \Pi_L(E_2)$$

This allows us to remove columns before the union, requiring less **data movement**

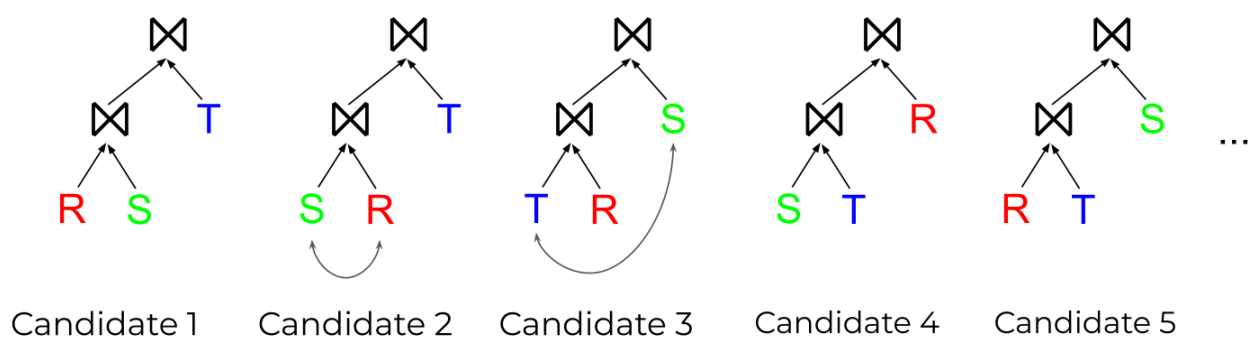


Figure 5.1: Example application of the rewriting rules. (Figure from lecture slides 08, slide 21)

5.1.2 Execution Model

There are a number of ways in which the execution can happen. More details can be found in section 5.2.6

5.1.2.1 Materialization Model

Here, each operator processes its entire input all at once and then emits its output all at once.

This is ideal, if the intermediate results are not much larger than the final results, as very little overhead is created. The efficiency of this approach however is more limited the larger the intermediate results are.

5.1.2.2 Iterator Model

Here, each operator is an iterator, it takes as input a set of streams of tuples (each of which provides a `next()` function) and outputs a stream of tuples, that can in turn again be accessed using a `next()` function. To get a result, we run `root.next()` again and again, until no more data is available to be pulled in.

This is a **Volcano Mode**, data flows from bottom to top.

This method has the benefits of there being a single interface for all operators, thus we don't (need to) know what the previous operator did. Furthermore, iterators are easy to implement and there are little to no memory overheads and the whole thing can be pipelined, parallelized and distributed.

On the other hand, method calls introduce a large overhead and suffer from poor instruction cache locality.

5.1.2.3 Vectorization Model

This method is similar to the iterator model, but instead of a single tuple, the `next()` invocation returns a batch of tuples.

We can also use vector instructions on the batch of tuple, which reduces overhead by requiring fewer function and instruction calls.

Of course, the limitation here is that the hardware needs to support vector instructions, though nowadays that is hardly an issue, since **AVX** instructions are supported on CPUs as old as the Sandy Bridge Architecture (e.g. Intel Core i7-2600K) for Intel (2011) and Bulldozer (e.g. AMD FX-8100) for AMD (2011).

5.1.3 Cost Model

A cost model is used to **estimate** the computation cost of a given physical plan, without actually running it.

Since the cost of running each operator depends on the input and output size of it, we can easily convert this into cost.

Thus, to estimate the cost, we need to know the runtime of an operator and the number of inputs and outputs. The first of these two aspects is of course very easily attainable, whereas the **cardinality estimation**, i.e. the estimation of the number of tuples an operator returns, is very hard.

To make this a bit easier, commonly a **selectivity** or **reduction factor** is used, which tells us how much of the input an operator returns.

Below some reduction factor estimators (we assume that the values are uniformly distributed)

Predicate	Reduction Factor	Predicate Example
<code>col = val</code>	$1 / \text{\#UniqueValues}(\text{col})$	<code>col = 2</code>
<code>col > val</code>	$ \text{max}(\text{col}) - \text{val} \div (\text{max}(\text{col}) - \text{min}(\text{col}))$	<code>col > 5</code>
<code>col < val</code>	$ \text{val} - \text{min}(\text{col}) \div (\text{max}(\text{col}) - \text{min}(\text{col}))$	<code>col < 5</code>
<code>col1 = col2</code>	$1 \div \text{max}(\text{\#UniqueValues}(\text{col1}), \text{\#UniqueValues}(\text{col2}))$	<code>t.name = s.name</code>

This table shows the overall concept for the cardinality estimation. For each predication, a reduction factor is determined, as each predicate "filters" out some tuples.

Thus, the resulting cardinality is $\text{max}(\text{\#tuples}) \cdot \prod_{r \in RF} r$, where *RF* is the **bag** of all reduction factors.

In all of the above estimators, we assumed that the values are uniformly distributed. This of course is not always true — there are many cases where data is heavily skewed — in which cases, histograms are used.

Two types of histograms are commonly used:

- **Equi-Width**: Here, the heights (tuple counts) of the buckets differ, as we fix the values for each bucket. This is useful for identifying hot-spots.
- **Equi-Depth** (or Equi-Height): Here, the width differs, because we want all buckets to contain the same number of tuples. The size of a range helps with cardinality estimates
- **Singleton** (or Frequency): Plots how often a single item appears in a table. This is very useful to compute the selectivity of queries

To enable quick processing of the cost function, the following metadata, or statistics, are typically stored:

Table statistics

- Number of rows
- Number of blocks
- Average row length

Column statistics

- Number of distinct values (NDV) in columns
- Number of nulls in column
- Data distribution (histogram)

Extended statistics

- Index statistics
- Number of leaf blocks
- Levels
- Clustering factor

System statistics

- I/O performance and utilization
- CPU performance and utilization

5.1.4 Search

We now know how to generate a set of semantically equivalent physical plans and how to estimate the cost for each of them. The question that remains is how to efficiently find the best plan.

Since real-world queries can be very complex, searching for the best physical plan can be hard. There are a few options, but we only covered a simple approach.

We can constrain the search space by only considering **left-deep join trees**, i.e. trees that have only leafs on the right hand side of each join statement. The rationale behind this is that many of these are fully pipelined plans and no intermediate results have to be written to temporary files. This is the concept **System R**, the first relational database used.

An example for a non-pipelined left-deep join tree is one involving sort operations. This brings the complexity down to $\mathcal{O}(n!)$, where n is the number of relations in the join.

- Enumerate join orders (different left deep trees)
- Enumerate plans for each operator (hash join, sort merge join, nested loop join, ...)
- Enumerate access methods for each operator (B+-Tree, hash table, sequential scan, ...)

This can be achieved using dynamic programming.

We then find the best 1-relation plan, then the best 2-relation plan by finding ways to join a relation with the best 1-relation plan, etc.

While this is still slow, it's faster than enumerating all possible join trees.

Later, systems started introducing heuristics to reduce the number of choices that must be made in a cost-based fashion.

Heuristic optimization transforms the query-tree using a set of rules typically improve execution performance:

- Perform selection early (reduces the number of tuples to process)
- Perform projection early (reduces the number of attributes to process)
- Perform most restrictive selection and join operations before other similar operations

Some systems only use heuristics, while others combine them with partial cost-based optimization.

5.2 Database Engines

5.2.1 Storage Management

To make database engines easy to use, some abstractions have to happen.

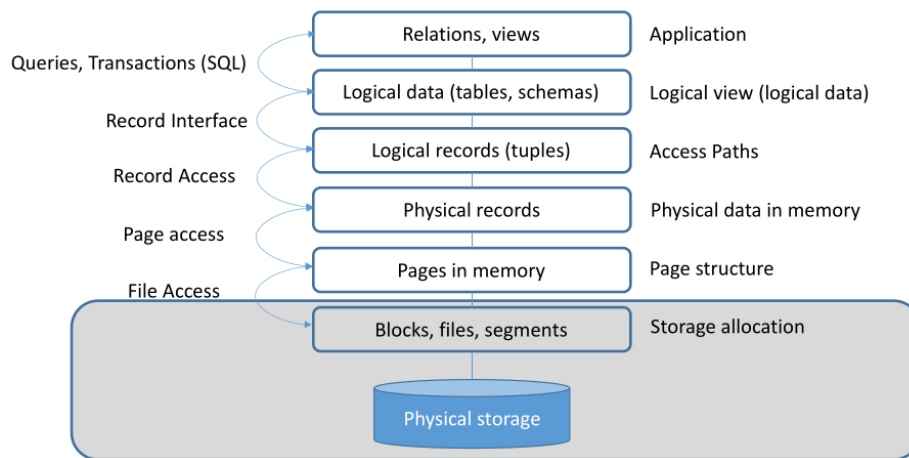


Figure 5.1: Overview of a typical abstraction for a database system (Figure from lecture slides 09, slide 3)

In the design of database engines it is commonly assumed that the data is stored on spinning disks, i.e. HDDs, with a high latency for seeks (random accesses) and that most of the DB is not in memory.

Very modern database systems may assume that a higher percentage of the DB is stored in memory due to the higher quantity of memory available, as well as lower access latencies due to the more readily available SSDs and high-speed NASes.

Some database systems operate similarly to operating systems in certain aspects, in that they manage their own processes and all of the memory to get the absolute maximum performance out of the systems.

5.2.2 Physical to logical storage

To the left, in figure 5.1, the relationship between the different storage abstractions are connected using crow's feet.

Since database tables often include large amounts of data, that would span many OS pages, thus database systems use blocks as opposed to OS pages.

These database blocks are structured as follows:

- **Header:** Contains the address and type of segment (that being index, table, etc)
- **Table directory:** Contains the schema of the table stored in the block
- **Row directory:** Contains pointers to the actual tuples stored in the block. Grows downwards (like the stack)
- **Free Space:** Well... it's free real estate, isn't it?
- **Row data:** The tuples stored in the block. Grows upwards (like the heap). The pointers in the row directory point here

Typically, database systems allow inserts of new tuples until there is about 20% of free space left. This space is allocated to allow for updates of existing tuples, as they may become larger. Thus, a larger storage footprint is used to prevent having to move all the data to a different location when the data block inevitably fills up, avoiding thrashing on the page.

Some systems even go a step further, only again unlocking a data block for writing if its used percentage falls below 40%.

Of course, as with any storage system, fragmentation becomes an issue. Compaction is only done when the block has enough space for an INSERT or UPDATE, but the space is not contiguous.

An UPDATE may require that a tuple is moved into a different block. In that case, the tuple is inserted into a new block and a pointer is stored in the freed space.

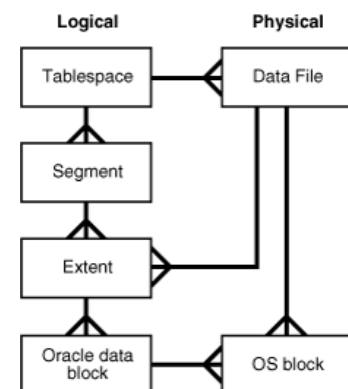


Figure 5.1: Entity relationship diagram, where the crow's feet imply one-to-many relationships (Figure from [Oracle Docs](#))

5.2.3 Record Layout

Each record (commonly called tuple) contains a header (which in turn contains validity flags for deletion, visibility information for concurrency control, bit maps for null values and more), and the non-null attributes, or pointer to those attributes.

Relational engines do not have to store the schema for the tuple inside of it because that is set globally for the entire table.

Schema-less systems however have to store that information because it can differ for each record.

5.2.3.1 Optimizations

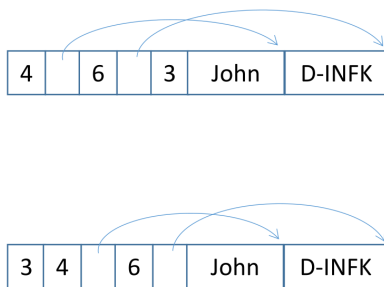


Figure 5.1: Record layouts in memory (Figure from Lecture 09, Slide 19)

It is crucial to store the data in a way that allows quick accesses and updates.

Each variable length attribute also has to store the length, so in a simple system we would simply store them together with the data. This however is not ideal for performance reasons (we have linear access time), thus we keep the fixed size part (attributes and lengths) at the start of the record and move the variable length part to the end, putting pointers after the length attributes that point to the end of the variable length region. Even more performance can be gained by reordering the attributes altogether, moving the variable length attributes to the very end and then applying the above optimization.

5.2.3.2 Data Types

- **Integers:** Most **DBMS** use the same format as C.
- **Real numbers:** Either in IEEE-754 (for variable precision) or using fixed point representations, which commonly are variable length, by storing all digits, plus where the decimal point is (they are NOT stored as strings)
- **Strings** and **BLOBs:** Store the length and the data
- **Time, Coordinates, points, ...:** System specific.

For large attributes, commonly instead of storing the whole tuple, only the fixed length attributes of the tuples are stored directly in the block, with pointers to the rest of the attributes, that may also lay outside of the current block. BLOBs may also be very large, taking up more than a single block.

5.2.4 Data in the blocks

The pages allocated to a given object (table, index, etc) are managed through lists, which are stored as part of the segment header

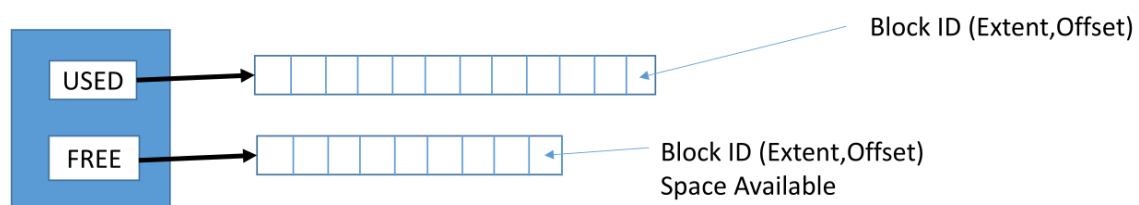


Figure 5.2: Free/used lists in databases (Figure from Lecture 09, Slide 24)

These lists can become a bottleneck, especially for transactions that result in modifications. There are some ways to improve the performance:

- We can use several free lists, so concurrent transactions can find free space in parallel, rather than having conflicts all the time
- Make the traversal of the list faster (sorting, etc)
- Make sure that holes can be found efficiently (store the available space in each page in incremental steps)

5.2.4.1 Slotted Pages

To find tuples within pages, slotted pages are used often. In slotted pages, each page has a header containing checksum, version, transaction visibility, compression information, utilization, etc.

Then, each tuple gets an id (typically a block ID and an offset).

The page maintains a list of the “slots” (each slot is assigned to one tuple and thus the number of tuples in the pages is further constrained by the number of pages available). The offsets, or sometimes pointers, are stored at the beginning of each tuple.

Advantages of slotted pages are:

- storing tuples of different sizes (and variable length tuples)
- Each tuple gets a permanent tuple / record id, that does not change
- When data doesn't change, space is used efficiently
- When data *does* change, we need to be careful

Considerations for when data *does* change:

- If a tuple is modified and becomes larger, the original space is used to store a pointer to the new location of the tuple
- If a tuple is deleted, we just remove the pointer
- If a tuple is modified and becomes smaller, leave the unnecessary space empty
- For insertion, we look for a page with enough free space and compact the page, if needed

The percentage free measure is used to avoid fragmentation caused by pages running out of space to accommodate updates for tuples that become larger. Thus, some space is reserved for growing the tuples, rather than for inserting.

Without this measure, the block can constantly go from FREE to USED and back and forth.

5.2.5 Row and Column Stores

5.2.5.1 Row Store

Here, as we have assumed thus far, tuples are stored with all their attributes together. This means that accesses to tuples is quick and easy.

This scheme is used by actual systems, but they also have many additional details added to improve management and speed.

A typical ideal application for a row store is Online Transaction Processing (OLTP). This is visible for example with banking applications, shopping carts, etc, where the operations mostly are on a single tuple.

However, for complex queries, the row store can become cumbersome.

5.2.5.2 Column Store

This is where the column store comes in, in which the data is stored by columns. This obviously speeds up operations on the columns, as the access is very quick.

Column stores also have the advantage that they present the data exactly in the representation needed for **SIMD** instructions such as **AVX**, which is now widely available on modern CPUs.

Modern database systems have started to adopt column stores to address the memory wall, since Column Stores are a more compact representation and cache lines are likely to bring the data we want to see, thus the cache utilization tends to be higher than with row stores.

Column store databases are today primarily used today for analytical databases and in-memory databases.

An alternative is a hybrid approach, called **Partition Attributes Across (PAX)** representation. There, each block contains several tuples, but it is organized as a column store.

5.2.6 Execution Models

We model execution as a tree of operators (as seen previously). At the root, we have the query results, at the leaves we have the tables. Typically, each operator has *at most* two inputs from lower layers (join operators have two, the rest typically has one).

We remember that we can use any subtree of a query as input for the higher parts of the tree, since we can “materialize” the result as a table.

5.2.6.1 Basic Execution Models

Each node (operator) in the tree operates independently, i.e. does input, putput, output.

Each operator can read their input tuples in one go, or one at a time and the same goes for the output. Some operators may also be *blocking*, i.e. no result can be issued until all operations are completed.

5.2.6.2 More advanced models

As a reminder, we already covered some aspects of three execution models in section 5.1.2.

5.2.6.2.1 Iterator Model

Some typical dataflow operators:

- **Union**: read both sides, issue all tuples (if they have the same schema)
- **Union without duplicates**: We first union, then we check for duplicates
- **Select**: We apply a filter function to the tuples
- **Projection**: For each tuple, we output only the specified attributes
- **Join**: Trivial nested loop join
- **Join with index on inner relation**: For all tuples, fetch matching tuple in S_2 = another operator

Advantages

- Generic interface for all operators
- Easy to implement operators
- Supports buffer management strategies
- No overhead in terms of main memory
- Supports pipelining
- Supports parallelism and distribution

Disadvantages

- High overhead of method calls
- Poor instruction cache locality

5.2.6.3 Pull & Push Mode

Definition 5.2.3 Pull Mode Intuitively like dynamic programming: Data is obtained by one operator through a function call to a lower operator. It is good for disk based systems and when data doesn't fit into memory and is also really easy to implement.

Definition 5.2.4 Push Mode This is the Pull Mode flipped on its head. While it *can* be faster, there need to be buffers everywhere and it is significantly harder to implement.

5.2.6.4 EXCHANGE operator

The EXCHANGE operator is useful for parallel processing, as it simply moves data from one place to another. This allows to parallelize a plan across one or more machines.

This can be implemented as a sort of “driver” on top of an operator, by calling `next()` on the operator a few times, collecting the results and sending them to the other side. The other side uses the data to respond to further next calls to the original operator.

5.3 Indexing

5.3.1 Access methods

Definition 5.3.1 Access Methods typically refers to the operations and data structures used for reading data from the buffer cache into the private space of the worker thread

The basic access method is **sequential scan**, where all pages of the table are read from beginning to end. Indexing is used to avoid having to scan the whole table.

There are many indexing techniques, but we will focus on three indexing techniques in this section, hashing, B+ trees and bitmaps.

Hashing is primarily used to randomize access to the data, finding data based on a key (hash and access concept) and both as an index for tables and used to implement operators.

B+ trees on the other hand are used to order data along the values of one attribute, but require traversal, unless we are looking for consecutive values. They are thus only used as an indexing mechanism.

5.3.2 B+ Trees

5.3.2.1 B-Tree

In B-Trees of order k , each node (except the root) has between $k \div 2$ and k child nodes. The root node has at least two children, unless it is also a leaf and any non-leaf node contains at exactly $k - 1$ keys.

Even if database systems say they use B-Trees, they do in fact use B+ trees.

5.3.2.2 Basics

B+ Trees are B-Trees, but the data (or the pointers to it) is at the leaves only, which are organized as a linked list. They are balanced trees, their inner nodes correspond to blocks and the leaf nodes correspond to blocks. The blocks are organized like slotted pages for variable length data (see Section 5.2.4.1)

The keys in the inner nodes may not correspond to the actual data and are instead used as separators. Typically, leaf nodes contain pointers to the tuples (value, key), where value is the actually indexed value and key is the pointer to the full data tuple, typically as a row id or tuple id. There are some systems that allow storing the data directly on the leaves.

Some systems create a B+ tree index by default for all tables and index the key. If there is no key, the engine assigns random keys and indexes them.

The height of a B+ Tree is given by $1 + \lceil \log_K(P) \rceil$, with K the number of keys per inner node and P the number of pages in the relation. This comes from typical logarithmic height, plus the leaf node (or the root node, if you like).

5.3.2.3 Clustered Indexes

An index orders the table by the attribute it is indexing, but the tuples in the table might not be ordered. This is where **clustered indexes** come into play: They force the tuples to be stored in the same order as the index indicates, thus, the table is physically stored in a sorted manner. This is typically done only for the primary key, and automatically done in systems that store data in the leaf nodes directly.

This eliminates random memory accesses caused by looking up ranges, since the tuples are stored in consecutive order, according to the index.

This is most useful for tables that are infrequently updated, since they require maintenance.

5.3.2.4 What to index?

This is a crucial question, as it can massively speed things up, but equally significantly slow things down.

B+ trees can use one or *more* attributes as the key to the index. If we have just one attribute, those are the values. If there are several attributes, it builds a composite key, which is useful when we are looking for combinations of values on certain attributes, or a table is searched by several attributes.

We then compare the keys lexicographically:

$$(a_1, a_2) < (b_1, b_2) \Leftrightarrow (a_1 < b_1) \vee (a_1 = b_1 \wedge a_2 < b_2)$$

Some values can also be left unspecified, but this is typically only done to the ones at the beginning of the key.

5.3.2.5 Composite Index

For a **SQL** statement such as

```
1 SELECT department_id, last_name, salary
2 FROM employees WHERE salary > 5000
3 ORDER BY department_id, last_name;
```

if we assume a composite index on `department_id`, `last_name`, `salary`, then the data is sorted by the three attributes in that order.

We can then perform a **full index scan**, where read the data from the leaves in sorted order and filter on salary. This avoids having to scan the entire table and the results is also already sorted.

5.3.2.6 Non-unique values

A B+ tree index can also be built on attributes containing also non-unique values.

This is a problem, because we cannot find all the duplicates with the basic design. To resolve this, we can repeat the key at the leaf nodes for every duplicated entry, or we can store the key once, but point to a linked list of all matching entries.

If the data is stored in the leaves, we append the tuple ID to know what tuple the entry refers to, because otherwise, they are all the same.

5.3.2.7 Operations

5.3.2.7.1 Direct Lookup

We traverse the tree from the root, then within each node, we use binary search to look for the correct entry. When we reach a leaf node, we return the corresponding pointer and the position.

5.3.2.7.2 Range Lookup

We return all tuples that match, by first finding the first tuple matching, the continue traversal until the last match is found. We can determine the last match easily because the tree's leafs are lexicographically sorted. Since the leafs are linked as linked lists, we can simply move along the linked lists, cutting down on processing time.

5.3.2.7.3 Inserting into the index

We first look up the corresponding leaf. Then, if there is space, we simply insert the value. If there is no space in the leaf, we split the leaf into two new leafs and insert the new item on the corresponding leaf.

All that remains to do now is to insert the separator on the parent node. If that node is full, we split the node in two and insert the separator into the parent node and propagate up to the root, if necessary.

5.3.2.7.4 Deleting from the index

We proceed by looking up the corresponding leaf, then we remove the entry from the leaf. If the leaf now is less than half full, we check the neighboring leaf

- If it is more than half full, we balance the two leafs
- If it is half full, we merge both leaves

For both operations, we update the separator in the parent node.

5.3.2.8 Concurrent Access

Indexes are heavily used while they are being maintained, thus creating conflicting, concurrent accesses.

A way to prevent this is using **lock coupling**, where we lock a page and its parent and move down the tree.

This prevents problems when the page must be split, however, this is not enough, if we have to go further up to apply the changes.

To prevent this, on every node we check if there is space for one more entry. If not, we split the node. This way, the split of a page never leads to propagating changes.

Alternatively, we can try lock coupling, and if we have to do a change that propagates up, we abort, release everything and start again from the top, but now lock the entire path.

5.3.2.9 Bulk Inserts

To create a B+ Tree index, we proceed as follows (the tree is created bottom up (from the leaves up))

1. Sort the data

2. Use the sorted data to fill one block after the other
3. Remember the largest value in each block
4. create the inner nodes by using the largest value in each block as separator
5. iterate upwards to the next level, until there is only one block left (the root)

If the blocks are filled completely, we get a clustered and compact tree. However, since we usually expect that data has to be updated, it is advised to leave some space in the each block to accommodate that without expensive propagating changes.

5.3.2.10 Optimizations

Reverse Index If the indexed attribute is a sequence and new items are constantly produced, we may encounter an issue where values next to each other go into the same block, leading to concurrent updates fighting to insert. Depending on how updates are handled, many copies could be produced.

Reverse indexing solves that issue by reversing the indices, meaning that consecutive indices go to different pages.

Many Keys Depending on attributes, many keys can become expensive to process. There are several possible optimizations:

- Replace separators that correspond to actual keys with shorter separators with the same effect (can be hard)
- Factor common prefix (this makes a lot of sense and is usually easy, too)

Ignoring rules We can also ignore the rules of the B+ trees and e.g. not merge nodes when they do not have enough data, delaying a merge (and instead rebuilding the tree periodically)

Variable Length Keys This is a bit of a problem. We can mitigate it by no longer forcing the number of entries to lay between $k \div 2$ and k , and using smaller values as separators, placing long values in leafs.

Making sure there is space We don't ever fill blocks to the max and make sure there is space on creation.

Depth vs Breadth For slow devices, prefer larger node sizes, thus shallower trees; For fast devices, prefer deeper trees.

5.3.3 Hashing

5.3.3.1 Hashing in Databases

Hashing is a very common operation in many systems, including databases. Many internal data structures, such as buffer caches and lock tables, are implemented as hash tables. Some operators also use hashing to speed things up, such as hash join and group by operators.

It can also be used as an indexing and partitioning strategy.

Hashing is on the other side compared to B+ trees when it comes to the trade-off between storage and compute, by using more compute, it can save on storage space requirements.

5.3.3.2 Hash Tables

There are many hashing functions with strong properties, but in databases, the hash function has to be very cheap, as it is used very often.

Perfect hash functions do exist, but they need a hash table that is as big as the cardinality of the attributes, so for 4 byte keys, it needs a 4 GB table. Thus, to save on storage, we use a hash table that is smaller than that. If we were to choose it too small, there would be too many collisions, if it is chosen too large, then we waste lots of space.

Collisions may also occur when the index key is not UNIQUE, and thus it could be that multiple tuples have the same key.

That may lead you to think, why not grow it if we have a collision. That is a valid idea, but growing hash tables is expensive, thus should be avoided.

Another shortcoming of hash indexes is that it only supports **point queries**, i.e. it only works for the primary key, but barely anything else.

5.3.3.3 Collisions

A way to handle collisions safely is to use **chaining**. When a collision occurs, we add another entry in a linked list. If these lists are reasonably short, then it is reasonably efficient. We can also already reserve space for the linked lists to speed things up, if we expect (many) collisions to occur.

Another option is to use **open addressing**. Here, when a collision occurs, we look for an empty slot in the hash table using some rule. Here, two ways of achieving this are covered in the course:

- **Linear probing**, where we go to the next slot(s) and place the attribute there
- **Cuckoo hashing**, where we use several hash functions and move to the next one, if we have a collision with the first, and so on

5.3.3.4 Extensible Hashing

When the hash table is full, in a basic system, we would need to create a new, larger hash table with more buckets (typically double the number) and then rehash all existing items. This of course is very expensive both computationally and in terms of memory accesses, as we do lots of random accesses, which leads to cache misses.

This is where extensible comes in. It is based on sharing buckets and unsharing them when needed. These hash table buckets are mapped into blocks and initially several buckets share the same block

Then, when a bucket gets full, we split the bucket and move entries as needed into a new bucket.

In **local sharing**, the size of a table can be doubled without immediately adding more space, allowing for more splitting. We simply increase the number of buckets and the degree of sharing and split the buckets as they become full.

Again, if we initially provide more space than needed, we can allow for the first growth to happen without disruption.

We want (ideally) to have two page lookups to access an item, so we typically have a lookup in the bucket directory and the data block and they both can grow independently.

5.3.3.5 Linear hashing

Here, a split pointer is used to indicate which bucket will be split in case of overflow. When it occurs, we chain the block that overflows, split the bucket indicated by the pointer and move the pointer.

This allows for a gradual increase in the size of the table and the idea is to gradually redistribute the data. We always split on the pointer, even if the overflow is elsewhere. The pointer will eventually reach a bucket with chains and when the bucket is split, the data is reorganized.

This has the advantage that the directory (i.e. the list of buckets) grows page by page, instead of doubling.

When all buckets have been split, start fresh.

5.3.3.6 In Postgres

PostgreSQL uses a variation of linear (overflow) hashing and extensible hashing (with doubling). Buckets are initially allocated in groups with powers of two. Overflows are captured in overflow pages (like in linear hashing). When a split occurs, we do not allocate all the pages, but a fraction of them.

These ideas can be combined in many different ways.

5.3.3.7 When to use

For memory structures:

- For shared resources (System Global Area)
- Database Buffer Cache for I/O
- Redo log buffer for recovery
- For large memory pools

Then of course, it can be used for a hash join, for the `SELECT DISTINCT` function and for aggregation (`GROUP BY`).

5.3.4 Bitmaps

Bitmap indexes are used for attributes with low cardinality (low number of distinct values), attributes that are retrieved by category and queries with low selectivity.

In addition, they are used to perform aggregation functions, such as `SUM` and to perform selection and operators over column stores.

Bitmap indexes are an alternative to B-trees and very space efficient.

Bitmaps work as follows:

- List all distinct values of an attribute
- For every distinct value (and for NULLs), create an array with 1 bit entries (i.e. 0 or 1)

- In the array corresponding to a given value of the attribute, set the positions in the array to one that correspond to tuples that have that value in that attribute.
- Now, each array indicates the tuples that have the value corresponding to that array.

5.3.4.1 Bitmaps as indexes

While a B-tree or hash index stores the row ID or tuple ID, a bitmap index only stores a single bit at a given position, which is indicating the of the tuple from the beginning of the table.

This means that to map a 1 in the beatmap to a tuple address requires applying a function that calculates the address. One way to achieve this is to layout the bitmap in page ID order. Then to map a bit position to a record id, we fix the number of records per page, lay the pages out such that the page numbers are sequential and increasing and then construct the row ID, page ID and slot number.

As a result, $\text{pageID} = \text{bit-pos} \div \text{\#records per page}$ and $\text{slot-number} = \text{bit-pos} \% \text{\#records per page}$

We can also compress bitmaps, in Postgres, this is done using run-length encoding.

5.3.4.2 Using bitmaps

Bitmaps are used in database systems to solve simple predicate selections, such as

```
SELECT NAME FROM STUDENTS WHERE Students.Department = D-INFK
```

or for multiple operations, we can perform operations directly on the bitmaps

```
SELECT NAME FROM STUDENTS WHERE Students.Department = D-INFK and Students.Degree = Bachelor
```

In addition, they are usually very sparse and long lists with just a few 1s compress very well, using run length encoding.

They are also often used for large tables in analytics as it is faster than other indexes for low selectivity queries and they are especially useful for expensive comparisons, such as for strings.

They can also be used to implement operators:

- NOT IN: get a bitmap with the results of the inner query, match it against a bitmap for the outer query, then retrieve the tuples
- COUNT: Count how many 1s appear in the bitmap
- JOIN: Match (AND operation) the bitmaps of the values of the two attributes
- RANGE predicates: Combine the bitmaps (OR operations) of all the values in the range
- ...

5.3.5 Bloom Filters

A bloom filter is an index that says whether a tuple with a given value is **NOT** in the original table. It acts as a filter, eliminating unnecessary accesses to the data. However, it can have false positives, i.e. it may state that a given tuple is there, but it is not in fact. Most importantly, it doesn't have false negatives, which is good, because this means we don't accidentally miss any tuples.

A bloom filter works by creating an array of size m , with values hashed using k hash functions. Each hash point points to a position in the array that is set to 1. Then, to check if an item is in the data, we check all the k hash positions for the value and if they all are all 1, then the item is in the data and if one (or more) is 0, then the element *definitely* isn't in the data.

The bloom filter's error rate depends on the number of hashes (k), the size of the array (m) and the number of items being hashed (n)

5.3.6 Zone maps

There are some systems that do not use indexes, or very sparingly. Instead, they keep metadata about the blocks and store them in memory and use it to check if the data in the block is needed.

This is common in column-store engines such as DuckDB and Snowflake. This metadata can be as simple as min/max for each column, or complete stats, bitmaps, precomputed aggregates, or even bloom filters.

5.3.7 AR Trees

Adaptive Radix Trees (ARTs) are mainly used to ensure primary key constraints and to speed up point and highly selective queries (selectivity $\geq 0.1\%$). It can also be manually created using CREATE INDEX and are automatically created for columns with a UNIQUE or PRIMARY KEY constraint.

The above applies to DuckDB, which is often similar to Postgres. ARTs are not balanced and the internal node size is modified to accommodate the data distribution, thus not requiring nodes to have fixed sizes, which would contain empty pointers.

The internal node size however typically isn't *entirely* variable, but often comes in four different sizes, facilitating traversal. Each node stores the keys and pointers to the next node to follow for a given key.

Common optimizations includes path compression, where particularly with long keys, some nodes could potentially only contain a single pointer. These nodes can be collapsed and removed to make the tree smaller (i.e. the pointer in the parent is updated to directly point to the data instead). If required by insertions, nodes can be added as needed (akin to B+ trees).

For speed, ART require indexed values to be binary-comparable. This can become an issue with data types such as signed numbers (extra bit for sign), strings (termination character) and nulls (must be mapped to a non-value sequence). Thus, the data needs to be transformed to be able to index it and search the tree.

5.3.8 R Trees

R-Trees are typically used for geospatial datasets, where searching for spatial relationship is expensive (requiring a full table scan).

R-Trees create indexes on bounding boxes as opposed to points in 2- or 3D space. We can locate the smallest bounding box where a tuple may lay by traversing the index and looking up the leaf to see if the point is there (or to look for a near neighbour).

For more information, see [Wikipedia](#)

(yes, they actually just linked to Wikipedia, so it probably isn't very exam-relevant)

5.3.9 Other Optimizations

For groups of data that don't change often, we can compute statistics on insert, such as `sum`, `avg`, `min`, `max`, etc. These aggregates can be used to check whether data needed is within that group. This also enable computing (some) aggregates without reading the entire table and is commonly used in data mining and data cubes.

A concept that is not used in DB engines, is count min sketch. It can be very useful in data streaming engines and large scale data processing. What this does is it computes statistics on how many times a given value was seen in a list or stream of data and is a variation of the bloom filter, but with several arrays and counting. The overestimation in count is bounded by the size in the matrix.

6 Query Processing

In this section, knowing the time complexities by heart is very important for the exam. They are highlighted **like this**.

6.1 Sorting

Sorting is crucial in DBMS due to the possibility that the user requests sorted data using an ORDER BY clause. In addition, it can help speed up some operators.

Something like Quicksort is fast, but creates many random accesses, which slows things down. Thus, sorting data as it is loaded into memory is beneficial, for which typically (a variant of) merge sort is used.

6.1.1 Merge Sort

When we sort each page on load into memory, we then have to merge all pages together, or more precisely, combine elements into pages in correct order. For that, we keep pointers to each pair of frames, then we first take first element of either the first or second frame, depending on which one comes first in the order, and copy it into the empty frame. We apply the same again to fill up the empty frame (to a threshold or fully). When the frame is full, we create a second one and link it.

We do this for all pairs, then apply the same procedure to each sorted frame group, repeating this until we have a unified, sorted set of frames.

If there is ever just one frame (group) left, we add an empty one implicitly and proceed normally.

The cost for this is for the 1-page sorted runs (i.e. sorting the original pages) is a single pass (read + write pass).

The total cost is given by $2 \cdot P \cdot N$ I/O operations, with P the number of passes / phases (i.e. how many merge steps there are) and N being the number of frames.

For Two-Way Merge Sort, the number of passes, P is given by $P = \lceil \log_2(N) \rceil + 1$, so the total cost to sort is $2N(\lceil \log_2 N \rceil + 1)$

The primary goal for sorting is of course efficiency. Primarily we want to reduce the amount of extra RAM used, the disk I/O (as that is slow) and trying to reduce random accesses in favour of sequential accesses. And of course, reducing CPU cost is still a concern.

We can increase the number of frames to B for the sorted runs, and then perform a $B - 1$ -way merge.

I/O cost for merge sort

(the 2 comes from the fact that each pass reads and writes N pages)

$$2N \cdot P \text{ with } P = (1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil) \text{ the number of passes}$$

If for instance, we want to sort $N = 600$ pages with $B = 50$ buffers, pass 0 creates $\lceil 600/50 \rceil = 12$ runs, which fits in a single merge pass because $12 \leq B - 1$, which is derived from the logarithm. Thus we have 2 passes and the cost in I/Os is $2 \cdot 2 \text{ passes} \cdot 600$.

6.1.2 Replacement Sort / Heap Sort

It has the advantage that it can produce longer runs with the same amount of memory, on average $2B$ (instead of B) with B frames of memory. It is used as the sorting algorithm in pass 0 above.

It works as follows:

Algorithm 4 Replacement Sort

```

1 procedure REPLACEMENTSORT( $R$ )
2   Read  $B - 1$  pages of relation  $R$ , store in a heap (priority queue)
3   for every page do
4     if Top of heap is smaller than the end of the sorted run then
5       Continue with next iteration
6     else if No element is left in memory that is larger than last element of current sorted run then
7       create new run
8     else
9       load page, remove smaller records and place them on the current sorted run, until a frame is freed.
```

Informally, it is searching for an always larger value once it has committed to a certain value. It picks the first value to be the lowest value in the current frames.

A new run is started if all values in the loaded frames are deferred or used already.

This means that after the sort run is complete, a merge is still needed.

6.1.2.1 Performance

Best case: N elements in a run (thus a single run, data is already sorted)

Worst case: $B - 1$ elements in a run (reversed data)

Average case: $2B$ elements in a run.

In any case, the cost of sorting is $2N(1 + \lceil \log_{B-1} \lceil N/E \rceil \rceil)$, with E the number of elements ($2B$ in the average case)

Thus, Quicksort is faster, but longer runs often mean fewer passes, saving time.

6.1.3 Sorting with B+ Trees

If a table has a B+ Index on the sorted column, we could retrieve the records in order by traversing the leaf pages. In case the B+ tree is clustered, this is a good idea, if it is not, then it could be very bad (due to random accesses).

The operating principle is very simple, abusing the linked lists in the leafs of the B+ tree, leading to each page being fetched just once, instead of one page fetch (= I/O) per record, if the data is not clustered, we have one I/O per record, which is bad.

6.2 Selection

Two terms important here are *logical selection*, which describes **what** we want to select and *physical selection*, which describes **how** the algorithm or procedure works that actually retrieves, or filters, the data.

The options include an *file scan*, where we scan the entire file and thus the I/O cost is $N \div P_F$, where N is the number of records in the relation and P_F the number of records per page. Alternatively, we can use *index scan*, where we use an index to retrieve the matching rows. The cost then of course depends on the index used and if said index can even be used to generate the results needed. We will cover that in more detail now.

6.2.1 Index Scan

- **Hash Index:** $\mathcal{O}(1)$, we read the bucket and possibly the overflow buckets. Can only be used for equality predicates
- **B+ Tree Index:** $\mathcal{O}(\log_F(N) + X)$ (or simply $\mathcal{O}(h + X)$, with h the height of the tree), with F fanout, N the number of leaf nodes and X the ratio of number of selected tuples and tuples per page. X can be up to 1 per selected tuple with an unclustered index, thus N for all. Optimization: we could sort the RIDs.
- **Bitmap Index:** $\mathcal{O}(\text{size of bitmap index}) + X$, X depends on clustering again

The I/O cost for B+ Tree Index Scan is **tree height + #leaf pages + #file pages**. Note that the number of leaf pages and file pages are multiplied with the selectivity of the predicate and are given by (for $\text{cnt}(R') = \text{cnt}(R) \cdot S$, with S the selectivity of the predicate):

- #leaf pages: $\text{cnt}(R') \div (P_L \cdot F_L)$, with P_L the #records per leaf page and F_L the fill factor for the leaf.
- #file pages: $\text{cnt}(R') \div P_F$, with P_F the number of records per file page.

Example 6.2.1 Computation example Given a relation R with $N =$ one million records. There are 100 records on a page and we have a B+ Tree with the data entries $\langle k, \text{rid} \rangle$ and it has a capacity of 500 data entries on each leaf. It also has 3 internal node levels and a fill factor of 0.67.

Then the cost is computed as follows:

- 3 internal nodes to parse, $\lceil \log_F(N) \rceil$
- Number of result records = $1,000,000 * 1\% = 10,000$ (this is the selectivity)
- Number of leaf pages pointing to the results records = $10,000 / (500 \cdot 0.67) = 30$
- Number of pages in the heap file that hold the result records = $10,000 / 100 = 100$

Then, the total cost is $3 + 30 + 100 = 133$

6.2.2 Index Matching

The predicates in the queries can contain comparison operators, as well as AND and OR, allowing conjuncts and disjuncts.

Index Matching is the process of choosing the proper index for the task. For conjuncts only, we can use:

- **Hash Indexes.** Constraints: Only equality operations and predicate has to contain all index attributes
- **B+-Tree.** Constraints: The attributes in the predicates need to be prefixes of the search key

Here, the indexes need to match **every** predicate in the disjunction. For instance, if we have a hash index on A and a B+ tree on B , and the query is $A = 7 \text{ OR } B > 5$. In that case, we can either file scan or use both indexes (we fetch the RIDs and take the union)

However, index matching is a hard problem.

6.3 Projection

For a basic `SELECT A, B FROM R` query, we scan the file and for each tuple output A, B . Obviously, that means that for N tuples and ratio of included tuples of Q , we have $N + N \cdot Q$ I/O operations. (N to read, $N \cdot Q$ to write the filtered tuples)

For a `SELECT DISTINCT A, B FROM R` query, we scan the file and eliminate duplicates before outputting. If this is also combined with a sort, we can prefer the sort-based approach and deduplicate after search, so we are projecting, then sorting and finally deduplicating using the sort method.

6.3.1 Duplicate Elimination

This is done using either hashing or sorting.

6.3.1.1 Hash-based approach

Assume we have B memory buffers. Then the hashing approach works as follows:

1. Project out attributes and splitting the input into $B - 1$ partitions using a hash function $h1$.
2. After that, all duplicates will be in the same partition. Since there most likely are further collisions, for each partition from (1), we do the following:
 - (a) If the partition fits into memory, then we are done with this step
 - (b) If not, we run step recursively run step 1 again on this partition
3. We return all keys from the hash table

A partition from step 2 fits into memory if $\frac{f \cdot R'}{B-1} < B$ (or approximately $B > \sqrt{f \cdot R'}$), where R' is the number of pages after the projection and f is a *fudge factor*, typically $f \approx 1.2$.

Cost of hash-based duplicate elimination

$\text{Cost}(R) = \text{cnt}(R) + \text{cnt}(R') + \text{repart}(R') + \text{cnt}(R')$, with $\text{repart}(R') = \sum_{i=1}^N 2 \cdot \text{cnt}(R')$, with N the number of times we had to re-partitioning the data to (typically) $B = (B - 1)^2$ partitions. The $\text{cnt}(R')$ is for dedupe. R' is the relation R after the projection. $\text{cnt}(R)$ is the number of pages and $\text{cnt}(R') = \text{cnt}(R) \cdot Q$, with Q the ratio of kept and total attributes in the relation.

6.3.1.2 Sort-based approach

A sort-based approach is also very easy to understand: We sort the records and then discard, on a run through the sorted records, the ones that are duplicates.

The cost here is $\text{Cost}(R) = \text{cnt}(R) + \text{cnt}(R') + \text{cost}_{\text{sort}}(R') + \text{cnt}(R')$. Broken down:

- $\text{cnt}(R)$ I/Os for the initial scan
- first $\text{cnt}(R')$ I/Os for storing the projected R (denoted R') on disk
- $\text{cost}_{\text{sort}}(R')$ I/Os to sort the remaining attributes (see 6.1)
- second $\text{cnt}(R')$ I/Os to scan the sorted result and eliminate duplicates

There is also an optimized version that uses a modified sort algorithm that during sorting projects out the attributes, with cost $\text{Cost}(R) = \text{cnt}(R) + \text{cnt}(R') + \text{cnt}(R')$. The first count is to read, sort and project; second count is to store the result to disk; third count is to eliminate duplicates.

6.3.1.3 Comparison

The sorting approach has the benefit of better handling data skew and that the result is sorted. The I/O costs are comparable if B is large, both algorithms need 2 passes over the data. For low numbers of records, the hash-based approach is likely cheaper

6.3.1.4 Index-based approach

Here, we only scan the index, and we don't visit the relation. The projection attributes need to be a subset of the index attributes. We can project distinct on A using a B+ tree on A, B . We then only apply the projection algorithm only to the data entries.

Then, if an ordered index contains all projection attributes as prefix of the search key, we first retrieve index data entries in order, discard unwanted fields and compare adjacent entries to eliminate duplicates (if needed).

If an index is smaller than a relation, then it is faster to scan the index.

6.3.2 Set Operations

6.3.2.1 Union / Union All

$R \cup S$ is the operation to perform. We can again use a sort-based or hash-based approach:

- **Sort-based:** We first sort both relations on all attributes and merge the sorted relations, while eliminating duplicates (except if UNION ALL)
- **Hash-based:** We first hash-partition both R and S . We then build an in-memory hash table for each partition R_p of R and probe with tuples in corresponding S_p of S and add to the output, if not duplicate. (Again only if not UNION ALL)

6.3.2.2 Difference

$R - S$ is the operation to perform. We can again use a sort-based or hash-based approach:

- **Sort-based:** We first sort both relations on all attributes and merge the sorted relations, while eliminating tuples from R that exist in S
- **Hash-based:** We first hash-partition both R and S . We then build an in-memory hash table for each partition R_p of R and probe with tuples in corresponding S_p of S and add to the output, if it does not exist in the hash table.

6.4 Aggregation

And again, we have the option to do this via sorting or hashing:

- **Sorting:** Sort on the attribute. Then scan the sorted tuples, computing running aggregate (such as max/min, average, etc). When a new group is encountered, then we output the aggregate. The cost is $\text{cost}_{\text{sort}}(R) + \text{cnt}(R)$. Alternatively, we can already compute the aggregates on the last step of sorting (the merge) to save some time. The limiting factor then becomes the sorting and the cost is $\text{cost}_{\text{sort}}(R)$
- **Hashing:** We hash the attribute and now each hash table entry is a group of all records with this value of the attribute. For each one of these groups, we compute the aggregate. The cost is $\text{cnt}(R)$. If the table is too large for memory, we use a two-step approach, as before. The cost is then very similar $\text{cnt}(R) + B$ for B tables.

6.5 Join

6.5.1 Simple Join

Algorithm 5 SimpleJoin Algorithm

```

1 procedure SIMPLEJOIN( $R, S$ )
2   for each tuple  $t_R \in R$  do
3     for each tuple  $t_S \in S$  do
4       if  $\text{pred}(t_R, t_S)$  then
5          $\text{add}\{t_R, t_S\}$  to the join output
  
```

Here, the max output size is $T_R \cdot T_S$, where T_X is the number of tuples for relation X .

The cost is obviously $\text{cnt}(R) \cdot \text{cnt}(S)$, with $\text{cnt}(X)$ the number of tuples in X .

6.5.2 Nested Loops Join (NLJ)

Algorithm 6 Nested Loops Join Algorithm

```

1 procedure NESTEDLOOPSJOIN( $R, S$ )
2   for each page  $p_R \in R$  do
3     for each page  $p_S \in S$  do
4       Join the tuples on page  $p_R$  with the tuples on page  $p_S$  (evaluate the predicate for all pairs)
5       Add matching tuples to the join output
  
```

The cost here is $P_R + P_R \cdot P_S$ I/Os, where P_X is the number of pages in the relation X . Furthermore, the DBMS should put the smaller relation on the outer loop to improve performance.

For multiple joins, we want to start with the smallest table (on the inside) and move up to larger ones, processing the largest one *last* (such that it is the outer loop)

6.5.3 Index Nested Loops Join (INLJ)

Algorithm 7 Nested Loops Join Algorithm

```

1 procedure INDEXNESTEDLOOPSJOIN( $R, S$ )
2   for each page  $p_R \in R$  do ▷ Outer loop
3     for each tuple  $t_r \in p_R$  do
4       Probe the index on the join attribute of  $S$  ▷ Inner loop
5       Add matching tuples to the join output
  
```

The cost here is $P_R + T_R \cdot C(I_S)$ I/Os, with P_X the number of pages in the relation X , T_X the number of tuples in said relation and $C(I_X)$ is the cost to probe the index I_X of relation X and depends on the clustering of data and type of index.

This operation needs three buffer frames. 1 for R , 1 for output and 1 to traverse I_S

6.5.4 Block Nested Loops Join (BNLJ)

We have B buffer frames, of which one is reserved for the pages of S and one is reserved for the output, thus the $B - 2$ blocks. Each of these blocks houses $P_R / (B - 2)$ pages of R , reducing search costs by only scanning S once for every block.

The cost here is $P_R + (P_R / (B - 2)) \cdot P_S$ I/Os. If however, R fits into memory, we have cost $P_R + P_S$ I/Os. We can reduce that further by building as second step (right after the first for loop) a in-memory hash table for the tuples in the R pages block. This reduces the cost of comparisons, however, we need $(B - 2) \cdot f$ memory now.

6.5.5 Block Index Nested Loops Join (BINLJ)

The cost here is $P_R + T_R \cdot C(I_S)$ I/Os. We sort the tuples in each buffer frame on the predicate key, which leads to tuples in R with the same key only requiring a single index search. In addition, tuples with “similar” values in R will likely match with keys on the same pages of I_S , which will already be in the buffer pool.

In practice, the difference between BNLJ and NJS is fairly stark.

Example 6.5.1 $P_R = 500$, $P_S = 1000$, 100 tuples per page and we have $B = 12$ frames. Then:

- $\text{Cost}(\text{NLJ}) = P_R + P_R \cdot P_S = 500 + 500 \cdot 1000 = 500,500\text{IOs}$
- $\text{Cost}(\text{BNLJ}) = P_R + (P_R / (B - 2)) \cdot P_S = 500 + (500/10) \cdot 1000 = 50,500\text{IOs}$

We only need very little more memory for BNLJ, with a big improvement in performance.

Algorithm 8 Block Nested Loops Join Algorithm

```

1 procedure BLOCKNESTEDLOOPSJOIN( $R, S$ )
2   for all  $B - 2$  blocks of pages of  $R$  do
3     for each page  $p_S \in S$  do
4       Join the tuples on page  $p_S$  with the tuples in the block
5     Add matching tuples to the join output

```

Algorithm 9 Block Index Nested Loops Join Algorithm

```

1 procedure BLOCKINDEXNESTEDLOOPSJOIN( $R, S$ )
2   for all  $B - 2$  blocks of pages of  $R$  do
3     sort the tuples in this block.
4     for each tuple  $t_R$  in  $B - 2$  block of  $R$  do
5       Probe the index  $I_S$  on the join attribute of  $S$ 
6     Add matching tuples to the join output

```

6.5.6 (Simple) Sort Merge Join (SMJ)

We first sort R and S on the join attributes with external merge sort (see Section 6.1). We then read the sorted files and merge. This is of course not great for performance, but if the relations are already sorted, we can skip the first step.

The cost (if there are **no duplicates**) is $\text{Sort}(S) + \text{Sort}(R) + P_R + P_S$ I/Os.

If there are duplicates (i.e. the keys are not unique, say we are joining on a column that doesn't have the UNIQUE constraint) then we need to move the pointer for the join back on the second relation to make sure we don't miss any tuples.

The cost now of course isn't linear anymore and worst case quadratic, $\mathcal{O}(\text{Sort}(S) + \text{Sort}(R) + P_R \cdot P_S)$.

Compared to BNLJ, its performance doesn't depend on the amount of memory available, but tends to be a tad slower than BNLJ, if BNLJ is provided with ample memory.

6.5.7 Optimized Sort Merge Join ((O)SMJ)

In this section, we are exploring how we can optimize sort and merge with extra memory.

Sorting can be optimized using replacement sort, the output is $\frac{P_R + P_S}{2B}$ sorted runs of length $2B$.

That of course means that the logical optimization is to do a K -way merge to merge the sorted runs and join the relations at the same time. If we have B frames, so we can do a $B - 1$ way merge, otherwise, we merge runs from each relation separately to reduce the number of runs until we have less than B sorted runs, then we perform the join.

The cost drops to $3 \cdot (P_R + P_S)$ I/Os, but we need memory such that $B^2 \geq \max\{P_R, P_S\}$

6.5.8 Hash Join (HJ)

This works by using a hash function that maps values into $B = P_R \cdot f$ buckets, with f being the Minister of Magic (up until, and including book 5) factor, also known as the fudge factor, which is additional hash table information stored alongside the tuples.

We first partition the table R into these buckets. Then, we probe the hash table for all tuples of S .

Ideally, we would want each bucket to contain exactly the same number of pages ($N/(B - 1)$ pages), but due to to hash collisions and duplicate keys, this is not likely to happen. To take care of this imbalance, we do another pass with a different hash function.

We should *NOT* use hash join if the hash table doesn't fit in memory because then it will become VERY inefficient.

6.5.9 Partitioned Hash Join (PHJ)

If the table doesn't fit into memory, then we can't use a normal hash join because that would be terribly inefficient.

Instead, we partition both tables into buckets and join the corresponding partitions.

The benefit of this approach is that in the third step, we are only operating on small numbers of records each, which can be done efficiently in memory.

What we ideally do in the above algorithm is to do *recursive partitioning*, i.e. as already *somewhat* described, we partition each partition again, if it has more than one page. This further reduces access times.

Algorithm 10 Partitioned Hash Join

```
1 procedure PARTITIONEDHASHJOIN( $R, S$ )
2   Partition  $R$  into  $RP$  partitions, using hash function  $h$  on the join key
3   Partition  $S$  into  $SP$  partitions, using hash function  $h$  on the join key
4   Join each partition  $RP$  with the corresponding  $SP$  partition using BNJL or building a hash table  $SP$  or  $RP$  in
   memory. This hash function should be different from  $h$ .
```

The cost for this is $3 \cdot (P_S + P_R)$, thus both OSMJ and PHJ are very similar in performance.

6.5.10 Conclusion

We can use all algorithms for equality joins (equi-joins), but if we have equalities over multiple attributes, then:

- PNL, BNL: work the same
- INL, BINL: build indexes on one, or all attributes of the equality conditions
- SMJ, HJ: Sort or hash on the combination of the attributes

For inequality joins:

- PNL, BNL: work the same
- INL, BINL: need clustered B+ tree index
- SMJ, HJ: don't work at all

7 Concurrency Control and Recovery

7.1 Transactions

Transactions are accesses to the database with the purpose of *modifying the data*. The focus is of course on correctness, recovery and state management.

Transactions typically are expensive compared to queries due to the focus on correctness.

This means we need to employ concurrency control (ensuring data remains consistent with transactions and queries running at the same time, definition of consistency, correctness, conflicts and problematic situations, as well as locking). We furthermore want to implement recovery, to ensure that data remains consistent even when unexpected failures occur.

7.2 ACID

ACID is the conventional notion of correctness (informal because it's an acronym):

- **Atomicity**: A group of operations must take place in their entirety or not at all
- **Consistency**: Operations should take the database from a correct state to another correct state
- **Isolation**: Concurrent execution of operations should yield predictable and correct results
- **Durability**: The DB needs to remember the state it is in at all moments, even when failures occur

Definition 7.2.1 **Database state** is the actual values stored in a database at a given point in time (in memory and storage).

Definition 7.2.2 **Consistent state** is a database state that is the result of *correctly* applying *operations* to the DB.

Definition 7.2.3 **Correct application** The model of correct executions of operations is sequential executions.

Definition 7.2.4 **Operations** The operations for transactions are inserts, updates and deletion of tuples.

Definition 7.2.5 **Consistency** The assumption of correctness of transactions is based on two things: **(1)** the user doesn't apply wrong transactions and **(2)** the DBMS has mechanisms to prevent incorrect modifications, such as violations of constraints, etc.

Definition 7.2.6 **Durability** ensures that the changes of a completed transaction are remembered by the system and can be recovered. This is often more strict and DBMS will keep the DB state and a history of all transactions.

The transaction history is captured in the DB log. The state at a given moment is captured in snapshots. The log and snapshots allow to go back and forward in the history of the database by undoing or redoing transactions from a given snapshot. Thus, very similar in concept to what `git` uses.

7.2.1 Atomicity

Definition 7.2.7 **Atomicity** dictates that a transaction only takes effect if it can be executed in its entirety, or in other words, a transaction has to be executed in its entirety or not at all. The issue with this is that violations are costly to correct.

Thus, atomicity requires isolation. If a transaction can see the intermediate state created by another transaction, then its input is not guaranteed to be consistent, implying that its output is not guaranteed to be consistent, either.

The DBMS get around this issue by isolating the transactions, such that they behave as if they were alone in the database. This of course means that we need to know when a transaction is completed so we can *commit* it, if there are no conflicts. If there are conflicts, we have to resolve them, rerun the transaction (note that this can be very expensive) or to prevent them altogether.

The isolation is enforced through concurrency control mechanisms to prevent conflicts. The canonical concurrency control mechanism in DBs is locking. We use locking policies to ensure that the transactions are isolated.

7.3 Transaction Model

Definition 7.3.1 Some terms for Transaction model:

- **Begin of transaction (BOT):** Often implicit
- **Commit:** Transaction has finished, database confirms to client when all changes of the transaction have been made persistent
- **Abort/rollback:** transaction is cancelled, all changes made already are rolled back
- $a <_T b$: a happens before b , is a *partial order*, which implies that a will be done before b .

In real systems, transactions are often associated with sessions and all SQL statements after an update statement are considered to be part of the transaction. This means they are explicitly managed, using a `BEGIN TRANSACTION` or `BEGIN` statement and ended with either a `COMMIT` or `ROLLBACK` statement.

So, we have the following statements:

- `COMMIT`: Transaction will not make any order
- `ROLLBACK`: Roll back all changes
- `SAVEPOINT <name>`: Create a save point / history point, used to roll back some changes and try some others in case of failure.
- `ROLLBACK TO SAVEPOINT <name>`: Roll back to a specific save point.
- `RELEASE SAVEPOINT <name>`: Delete a save point (doesn't delete the changes made before, just the log entry)

7.3.1 Operations

Below a list of very important syntax for the exam (where the subscript i denotes that this operation is part of transaction T_i):

- Read operations: $r_i[x]$ or $r_i(x)$ is an access to tuple x without modifying it.
- Write operations: $w_i[x]$ or $w_i(x)$ represents a write operation on the tuple x .
- Abort: a_j is an abort of transaction T_j
- Commit c_j is the commit of transaction T_j

Conflicting operations are two operations on the same item with at least one of them being a write operation, such as $r_1[x]w_2[x]$ or $w_1[y]w_2[y]$.

7.3.2 History

A history H is a partially ordered $<_H$ sequence of operations from a set of transactions. If two operations are ordered within a transaction, they are equally ordered in the history. If two operations p and q conflict, then they are ordered with respect to each other ($p <_H q$ or $q <_H p$)

7.4 Concurrency Control (CC)

The goal of concurrency control is to ensure the correct executions of concurrent transactions. The baseline for correctness is, as previously mentioned, serial execution.

Definition 7.4.1 Serial History A history H is serial if, for every pair of two transactions T_i and T_j that appear in H , either all operations from T_i appear before all all operations of T_j , or vice versa. A serial history with only committed transactions is correct by definition, because transactions are isolated and each transaction starts in a consistent state and leaves the DB in a consistent state.

Definition 7.4.2 Equivalent History Two histories are equivalent if and only if

1. They are over the same transactions and contain the same operations
2. Conflicting operations of non-aborted transactions are ordered in the same way in both histories

Definition 7.4.3 (Conflict) Serializable History A history is serializable if and only it is equivalent to a serial history.

A very simple way to check this is that a history is not CS if there are conflicting operations.

Theorem 7.4.4 Serializability A history is serializable if and only if its serializability graph is acyclic. The serializability graph is constructed from the history graph as follows:

1. The start node is the transaction the first operation.
2. Then, follow the operations, adding any new transaction to the tree and connect any further dependencies.

More visually, see the below graphics

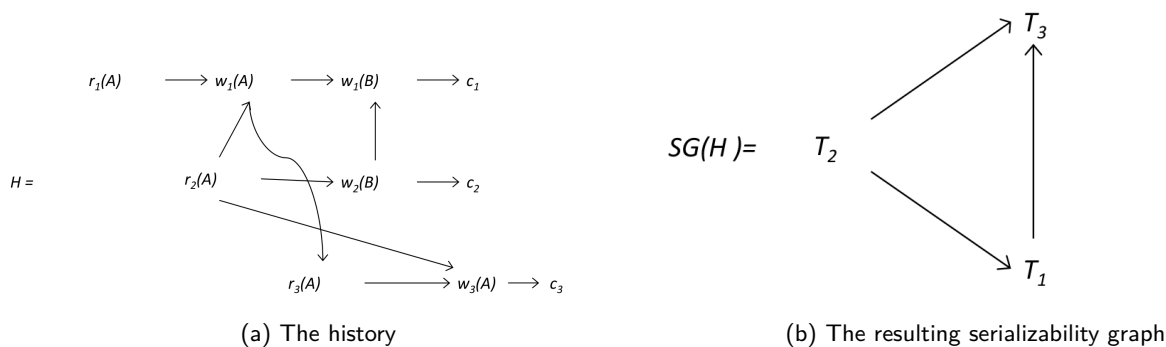


Figure 7.5: Building a serializability graph (Figure from lecture slides 19, slide 37 and 38)

7.5 2 Phase Locking (2PL)

This type of locking is widely used since many decades and is the canonical implementation of concurrency control. Of course, alternatives exist and the locking implementation has a huge impact on performance and scalability.

Concurrency control is implemented using locking, typically using a locking table. The locks cover tuples, tables, indexes, blocks and more.

Locks have different semantics that indicate whether they can be shared or are exclusive.

Compatibility matrix (S is shared lock, X is exclusive (for writing), NL is no lock, left for current, top for next state):

	NL	S	X
S	✓	✓	-
X	✓	-	-

Note that to acquire any sort of exclusive lock (even if it's just an *intention* to do so later on in the transaction), no lock can be held on that object and no other kind of lock can be acquired. For shared locks, as the name would imply, any number of threads can also acquire a shared lock on the object.

7.5.1 The protocol

1. Before accessing an object, a transaction must acquire lock
2. A transaction acquires a lock only once. Lock upgrades are possible
3. A transaction is blocked if the lock request cannot be granted according to the compatibility matrix
4. A transaction goes through two phases:
 - (a) Growth: Acquire locks, but never release locks
 - (b) Shrink: Release locks, but never acquire locks
5. At EOT (commit or abort), all locks must be released.

The reason for the two phases is that 2PL prevents histories where a transaction must be aborted due to concurrency control issues. An operation is thus not allowed unless we are sure it is safe.

Without the two phases, the following can happen: $L_1[x]w_1[x]U_1[x]L_2[x]w_2[x]U_2[y]L_2[y]w_2[y]$, etc, thus meaning that if T_1 now access y or x for read or write, the history becomes non-serializable.

The basic 2PL doesn't say anything about aborting or committing transactions. The issue is that if we are to undo the changes of T_1 , we would also undo some of the changes from T_2 , which would then in turn not be restorable anymore.

7.5.2 Strict 2PL

Strict 2PL combines 2PL with strict histories. This means that we enforce 2PL, but we also keep the read and write locks until the transaction commits or aborts. This results in serializable and strict histories. All DB engines implement strict 2PL.

7.5.3 Deadlocks Detection

In theory, we could use a wait-for graph, which is however expensive. For example, for $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_1$, we could abort T_3 to resolve the cycle.

In practice, no wait-for graphs are built. Transactions and queries have timers. If the timer expires, the transaction or query is aborted. Deadlocks are considered rare enough to prefer making mistakes every now and then (aborting a transaction that was fine), rather than implementing something as expensive as a wait-for graph.

Implications from this are for example that long transactions may block queries and that those queries may abort because the timer expired.

However, this is only common in poorly written applications.

7.6 Snapshot isolation

This was initially used by Oracle and is a form of Multi-Version Concurrency Control (MVCC).

MVCC applies serializability, but writes generate a new version and reads read from a concrete version of the DB. This means that the whole DB can be implemented (almost) entirely without locks.

When a transaction starts, it receives a timestamp T . Read operations only have access to items committed before T . All write operations are carried out in a separate buffer (shadow paging) and only become visible after a commit. When a transaction commits, it checks for conflicts. If there are, the first-committer-wins rule is applied, i.e. if we try to commit and there are conflicts, this transaction has to rerun (or be aborted).

However, there are scenarios, such as $r_1[x]r_1[y]r_2[x]r_2[y]w_1[x]w_2[x]c_1c_2$, where the history is not serializable: $T_2 \rightarrow T_1 \rightarrow T_2$. Depending on the application, this can still be correct (if this is for a ticketing system), but can also be disastrous (for e.g. a bank).

If done properly however, it has significant performance advantages, as well as in distributed settings with replication.

Note that a transaction cannot commit when a previous transaction has modified an object that this transaction has also modified, regardless of whether or not this transaction has read the object or not. Thus, the commit checks only check for writes, not for reads.

7.7 Recovery

Recovery has to make sure that, even in the case of failures, the changes from transactions that have not completed are not visible and those of committed transactions are recorded and visible. We define undesirable situations and classify histories according to these situations. Furthermore, we want to define how histories can avoid such situations.

7.7.1 Recovery procedures

- R1 Undo all changes from transactions
- R2 Redo the changes from committed transactions
- R3 Undo the changes that remain in the system from active transactions
- R4 Read consistent snapshot from backup, and, if available, apply log

R1 is used for abort/rollback of single transactions, R2 and R3 are used on system crash (where we lose the memory, but disks survive) and R4 is used for recovery from a system crash with disk loss.

This means that we can undo the changes of an active transaction (we keep a copy of the value before it was modified; Shadow pages is a mechanism that would support this), we can redo changes made by committed transactions (we keep a persistent record of all the changes made by committed transactions, this why there is a redo log) and we have a consistent snapshot to start with (either move forward by taking a snapshot and apply committed transactions, or move backward by taking the current state and go back to a consistent state by undoing changes from transactions that should be there).

7.7.2 Undo - Redo

The changes of an aborted transaction are undone typically by restoring the **before image** of the modified value.

Changes are redone by restoring the **after image** of the modified value.

Databases typically log transactions by keeping before and after images of their changes.

7.7.3 Recovery on histories

At exam: They like to ask about these, so be sure to understand this properly. Same goes for Conflict Serializability for histories (see 7.3.2)

A transaction T_1 reads from another transaction T_2 if T_1 reads a value written by T_2 at a time when T_2 was not aborted.

We have:

- **Recoverable** (RC) history: If T_i reads from T_j and commits, then $c_j < c_i$ (same goes for aborts)
- **Avoids Cascading Aborts** (ACA) history: If T_i reads x from T_j , then $c_j < r_i[x]$
- **Strict** (ST) history: If T_i reads from, or overwrites, a value written by T_j then $c_j < o_i[x]$ or $a_j < o_i[x]$, with $o = r$ or $o = w$ (o is an operation)

7.7.3.1 Recoverable

There is no need to undo a committed transaction because it read the wrong data and they commit in their serialization order.

Suppose we have the following sequence: $\dots w_2[x]r_1[x]c_1\dots$. If T_2 aborts, then T_1 has read invalid data and since T_1 has committed, the data has returned to the user. Recovery now involves correcting the application that has processed the data and might have acted upon it, which typically is a very expensive procedure. This is avoided by making sure that T_1 does not commit until T_2 commits and if T_2 aborts, then we can safely abort T_1 , since the results have not been returned to the user yet.

7.7.3.2 ACA

Aborting a transaction does not cause aborting others and transactions only read from committed transactions

Suppose we have the following sequence: $\dots w_2[x]r_1[x]a_2\dots$. When T_2 aborts, we have to abort T_1 because it has read invalid data (the change made by T_2 , which will be undone as part of rollback of T_2). If those cascading aborts happen regularly, the performance will degrade and possibly further transactions will be aborted. This is avoided by making sure that uncommitted data is never read (*read committed*).

7.7.3.3 Strict

Undoing a transaction does not undo the changes of other transactions. Transactions further don't read or overwrite updates of uncommitted transactions.

Suppose we have the following sequence: $\dots w_2[x]w_1[x]a_2\dots$. If we undo $w_2[x]$, we would also remove the changes made by T_1 . Thus, undoing T_2 implies aborting T_1 . Like in ACA, non-strict execution leads to cascading aborts of transactions. This is avoided by not letting a value be read or updated unless it is committed.

7.7.3.4 Why recoverability matters

Fixing errors with recovery typically are very expensive:

- Not RC: running my program issues a report, but the data that I got was later removed because a different program aborted $\rightarrow$ report is invalid
- Not ACA: Thrashing behaviour when transactions keep aborting each other (my program slows down, or makes no progress at all because of other programs)
- Not Strict: recovery after a failure becomes very complex (or impossible)

7.8 Recovery Procedures

Definition 7.8.1 Recovery Manager A recovery manager implements the recovery procedure, which depends on how the system functions. It may or may not require both, one or none of UNDO and REDO.

7.8.1 From the buffer cache's perspective

- **Steal Policy:** An uncommitted transaction is allowed to overwrite the changes of a committed transaction in persistent storage. This logically means that we need to be able to undo transactions.
- **Force Policy:** All changes made by a transaction must be in persistent storage before the transaction commits. This requires flushing all blocks with updates from the transaction and if not in-place, it will require being able to redo the transaction.
- **Steal/No-Force:** UNDO/REDO, is the most common approach

If for updates tuples are locked, blocks may still be updatable. Thus if the block is copied to storage, it is possible that some changes are committed while others are not. If failures occur, there is no guarantee that the data in storage is 100% consistent, thus requiring undo and redo.

7.8.2 Recovery Procedure for UNDO/REDO

- **READ:** simply read the value from the block in the buffer cache
- **WRITE:** Create log entry (before and after image), append it to persistent log and write the after image to the block on the buffer cache.
- **COMMIT:** We write a persistent log entry indicating that the transaction has committed
- **ABORT:** For all updates, we restore the before image using the log entry

The Procedure:

1. Start from the end of the log and work backwards
2. Keep a list of UNDONE items and another for REDONE items
3. Procedure terminates when all items are in either the UNDONE or REDONE list or we reach the beginning of the log.
4. For each log entry, we look at the data item x being accessed and if x is in none of the two lists:
 - If the log entry is of a committed transaction, apply the after image, add x to the REDONE list
 - If the log entry is of an aborted transaction, apply the before image, add x to the UNDONE list

The procedure ignores the data stored on disk as it could correspond to uncommitted transactions, it only takes it as starting point for the recovery. An alternative way of thinking about the procedure is as follows: This is not the way it's

Algorithm 11 Undo-Redo Recovery

```

1 procedure UNDOREDORECOVERY
2   for each item in the DB do
3     Find last committed transaction modifying the item and REDO it
4     if no committed transaction modified item then
5       Find the first aborted transaction that modified the item and UNDO the modification
6     if no transaction has touched the item then
7       Value is correct (we assume that we start from a consistent state)
```

done in practice, because there typically are many more items than log entries.

Advantages Only forced I/O are log records, buffer cache manager has a lot of freedom (no flushing dirty pages, I/O on data is minimized and only triggered for replacement policies) and we allow writing dirty data to disk, simplifying buffer management.

However, recovery is more complicated and takes more time, but normal operations are only minimally affected. Furthermore, queries are not affected since there is no forced I/O of data. Transactions are affected, because we need to write every operation to the log.

7.8.3 UNDO, no REDO

- READ: simply read the value from the block in the buffer cache
- WRITE: Create log entry (before image), append it to persistent log and write the after image to the block on the buffer cache.
- COMMIT: Flush all dirty values modified by transaction if still in cache. We write a persistent log entry indicating that the transaction has committed
- ABORT: For all updates, we restore the before image using the log entry

The Procedure:

1. Start from the end of the log and work backwards
2. Keep a list of UNDONE items
3. Procedure terminates when all items are in either the UNDONE list or we reach the beginning of the log.
4. For each log entry, we look at the data item x being accessed and if x is not in the UNDONE list and the transaction is aborted, we UNDO the changes by using the before image, adding x to the UNDONE list

It relies on the fact that all committed values are in persistent storage and therefore, they have not been lost.

However, it has forced I/O on all dirty blocks touched by the transaction when it commits, log records no longer need to include after images, reducing log record size and the recovery procedure is shorter because it only involves undoing aborted transactions.

However, this trade-off does not pay off in practice, as we still need to write the log with every update and while it requires smaller log records, it forces I/O on data blocks, which are often much larger. And finally, flushing the buffer cache interferes with its operations.

7.8.4 no UNDO, REDO

- READ: If the transaction did not write the item before, read the value from the block in the buffer cache. Otherwise, read the value from the temporary buffer
- WRITE: Create log entry (after image), append it to persistent log and write the after image to the block on the buffer cache.
- COMMIT: Apply all updates in the temporary buffer to the actual blocks. We write a persistent log entry indicating that the transaction has committed
- ABORT: Discard the temporary buffer

The Procedure:

1. Start from the end of the log and work backwards
2. Keep a list of REDONE items
3. Procedure terminates when all items are in either the REDONE list or we reach the beginning of the log.
4. For each log entry, we look at the data item x being accessed and if x is not in the REDONE list and the transaction is committed, we REDO the changes by using the after image, adding x to the REDONE list

It relies on the fact that there are never dirty blocks in the buffer cache. All data there is committed and is the last committed version.

It has forced I/O only on the log records and they are also smaller. Furthermore, the recovery procedure is shorter, only involving redoing the last committed transaction that touched the item. It is similar to snapshot isolation.

7.8.5 No UNDO, no REDO

This procedure is rarely used in conventional DBs. It does not require a log. However, it requires to be able to write all changes made by a transaction to persistent storage in **a single atomic action**. It relies on directories to keep track of the changes.

- READ: If the transaction did not write the item before, use current directory to find the latest committed copy. Otherwise, read the value from the shadow directory of that transaction to find the updated copy
- WRITE: Write to a buffer and add a pointer in the shadow directory for the transactions

- COMMIT: Create a full directory by merging the current one and the shadow directory of the transaction. Then, we swap the pointer indicating the latest committed directory.
- ABORT: Discard the buffer(s) and the shadow directory

There is no recovery procedure here, which is the exact point.

It is not used in practice, although some ideas can be applied partially. One of the reasons for that is expensive pointer indirections for accesses and requires garbage collection and moves data all the time, which poses a challenge for clustered indexes or hash clustered tables.

7.9 Implementation of Recovery

The most common implementation of these ideas is Write Ahead Logging (WAL). There, we separate the storage used for the log and data. The log contains enough information for whatever policy is chosen and log records corresponding to a change in the DB must be written to the log before changes to the data in the buffer cache are flushed to permanent storage. The COMMIT record in the log is used to mark the end of a transaction.

This is typically used to implement UNDO/REDO (on 2PL-based systems) or no-UNDO/REDO (on Snapshot Isolation (SI)-based systems)

7.9.1 Checkpoints

A log would grow forever if we don't do anything. Storage would then become an issue and recovery time would increase. Instead, Checkpoints are used. At a checkpoint, we do the following:

- Push all dirty blocks to the disk
- Push all the logs in the log buffer to disk
- We now have an active translation table and dirty page table
- We mark the log with a checkpoint label and flush it to the log

Then, recovery happens from a checkpoint instead from the beginning. However note that a checkpoint is not necessarily a consistent copy of the DB.

7.9.2 Recovery with WAL and checkpoints

Also called ARIES style recovery, if we use WAL, the recovery with a checkpoint works as follows:

Algorithm 12 WAL checkpoint recovery

```

1 procedure WALCHECKPOINTRECOVERY
2   Find latest completed checkpoint in the log
3   while Traversing the log to the end do
4     identify active transactions at time of crash and dirty pages that may not have made it to disk at time of
       crash for each element
5   Apply all updates (REDO) starting from the log entry matching the lowest SCN in the dirty pages
6   UNDO all transactions that were active at the time of the crash

```

8 Vector Search

Vector search is for example used in Semantic Search and of course search engines. It is also used to find relevant data to augment an LLM prompt (Deep Research or whatever that type of feature is called).

We have at least four metrics for the quality of results:

- **Recall**: relevant items retrieved by the vector search (R) divided by all relevant items (T), so $R \div T$. The closer to 1, the better
- **Cosine Similarity** S_C : The cosine of the angle between the two vectors. Values range from -1 to 1 and the closer to 1 , the better (higher = better)
- **Cosine Distance** D_C : defined as $1 - S_C$, values range from 0 to 2 and the closer to 0 , the better (lower = better)
- **Euclidian distance**: The norm of the vector between the two vectors. Values are from 0 to ∞ , the lower the better.

$$S_C(A, B) = \frac{A \cdot B}{\|A\| \cdot \|B\|}, \text{ a}$$

8.1 Trees/Clustering

Since nearest neighbour search (NNS) is not viable for large scale datasets (which they typically are), we use clustering algorithms such as K-Means to find K cluster centroids. We then assign to each of the centroids the vectors that belong to this cluster

Then, for search, we can check which cluster(s) the query vector is closest to. Due to the low number of centroids, this is comparatively cheap to do. We can then search all the vectors in the corresponding clusters, which is much cheaper due to the much lower number of vectors to search. Still, Vector Search is a very expensive operation due to the large number of vectors to be searched, even if the search itself is $\mathcal{O}(n)$, with n being the number of vectors to search through.

On insert, we search the the closest centroid and simply assign the vector to that cluster.

8.1.1 Inverted File

A not (directly, as far as I can tell) covered, but very much relevant index is the IVF index (Inverted File). It works similarly to a normal clustered index, just that each centroid holds an *inverted list* of assigned vectors. The `nlist` parameter defines the number of coarse clusters. The larger, the shorter the lists are, the (typically and up to a point) faster the scans are, but training / constructing it takes longer and it uses more RAM. `nprobe` sets the number of items to retrieve on each query, so for instance, setting it to one, we will only probe (access / retrieve) the nearest centroid's list.

8.2 Hierarchical Navigable Small World Index (HNSW Index)

The name used in the lecture was HSNW (Hierarchical Small Navigable World) Index.

This is a layered graph concept, at insert, first all nodes are inserted into layer 0 and each is connected to $M \approx 3 - 4$ neighbours. Then, the edge count is $\mathcal{O}(N \cdot M)$. With $N = 100$ million and $M = 16$, we have $1.6B$ edges, with a typical memory footprint of $4 - 8GB$ for $100M$ 32-bit Edge IDs. Then, some nodes (typically about $N_{L_k} = N/M^k$ to layer k) are promoted to layer 1, connected to M nearest neighbours in $L1$ only. This process is repeated a few times until the number of nodes in the layer is low enough. This number depends on a number of factors, primarily how coarse grained we want the initial search to be.

Then, after building, we prune edges, in a process called RNG pruning. Here we prune an edge (u, v) if there exist edges (u, w) and (w, v) , such that $d(u, w) < d(u, v)$ and $d(w, v) < d(u, v)$. Important is that both conditions have to be met, as the goal is to remove the longest redundant edge from the graph. This is done using a greedy algorithm.

Higher values of `ef_construction` generally improves the quality of the index at the cost of slower index construction.

8.2.1 Search

We note that greedy search always makes progress in this index because the graph stays navigable.

HNSW index search works by starting at a fixed entry, then doing *one* greedy hop only. The hop goes towards the neighbour on this layer that is closer to the query. This is often done using Beam Search (see below) to find the K closest vectors.

After some number of hops, there are no more neighbours in layer LN that are closer to check, so descend one layer and repeat. Traversal of the layer costs $\mathcal{O}(M_L)$, with M_L the number of edges on layer L . Higher up layers have fewer edges, thus making traversal cheap.

The parameter `ef_search` controls the recall vs latency. Set to 1 it is approximately greedy, for `ef_search` $\gg 1$ means more effort, i.e. more vectors visited, but also increases recall.

8.2.1.1 Beam Search

This is another greedy search algorithm, which chooses the next node to visit from a beam of size N . The goal is to find the closest node of the query, with d the distance to the query.

The procedure is simple:

Algorithm 13 Beam Search

```

1 procedure BEAMSEARCH( $G$ , start_node)
2   Beam  $\leftarrow$  new beam data structure
3   Beam.add(start_node)
4   while beam in Beam do
5     Check distances of neighbours and add them to Beam
6     Truncate Beam to size max_beam  $\triangleright$  This is a tunable parameter
7   return closest

```

8.3 Hash Index

To measure the quality of **ANN** in comparison to **KNN**, we use $\text{recall@k} = |\text{ANN_result} \cap \text{KNN_result}| \div k$. This however is not always a good metric, as two vastly different results, one objectively worse can score equally well.

Alternative methods mentioned are RDE@k and TDK@k, but not elaborated any further.

8.3.1 Performance vs Recall Trade-off

This is a challenging task, primarily focusing on how many vectors we need to visit to achieve a user-specified target recall.

A few explored strategies are:

- **Uniform autotuning for all queries:** Using learned offline models (e.g. Google CloudSQL VectorAssist)
- **Different for each query:** A model predicts the search effort parameters for each query or index
- **Adaptive:** Decide to continue or stop early based on the current search state

8.4 In Databases

The benefit of supporting vector search in DBMS is that this makes it easy to use for the end user and can be optimized for large datasets.

8.5 Filtered Vector Search (FVS)

This allows us to query both structured and unstructured data. In concept, this is **SQL** + Vectors, sometimes plus joins, subquery expressions, multiple vector searches (on both sides of the join). Sometimes, the dataset isn't even a vector + tag stored in main memory.

Indices are typically built on unfiltered data. Thus, we need to think about the proper execution method. Suppose we are to retrieve K rows with filtering:

- **Pre-Filter:** Filter first, then run NNS, and we get the K rows. This is expensive if the filter is not selective.
- **Post-Filter:** We first run **ANN** search, from which we get K' rows which we then filter to get our K rows. This is a recall and performance challenge, because if $K' \gg K$, then we have a lot of wasted effort. If on the other hand $K' > K$ and thus $\sigma(K') < K$, then we have low recall.

So, if the selectivity of the filter is high, then pre-filtering is better, if it is low, then use post-filtering.

One way around this issue is to apply an *inline filter*, i.e. during **ANN** search, we do vector search, evaluate the stopping condition and apply the filter. This gives us K rows directly and a handy performance uplift to go along with it.

Sounds too good to be true? That's because it is. There are still some issues, namely if filtered out nodes don't participate in the navigation. In that case, the search stops if we reach such a filtered node, but there would be nodes that are closer to the query behind this node, which due to the removal of the filtered out node are now no longer accessible.

This means that we need to have the filtered nodes participate in the navigation, which increases the cost by increasing the number of needed distance computations.

Thus, *filters increase the search effort and difficulty of tuning search parameters*. Selective filters make it harder to find valid nodes.

This is where the Value-Vector-Correlation comes in. It captures the relationship between the **probability of satisfying the filters** and the **distance from the query vector**. It doesn't however always correlate positively and the correlation depends on the query. For instance, a query like "Item like a blue T-Shirt and price $\leq$ \$10K" is negatively correlated.

8.5.1 Hash-Tree Index

If we use a Hash-Tree instead of the HNSW index, with filtering the internal node navigation does not change, as the data vectors are only in the leaves, so we simply filter there.

8.5.2 Performance Challenges

The cost of a vector search is given by $f(n_d \cdot c_d, n_f \cdot c_f)$, where n_d is the number of distance computations, n_f is the number of filter evaluations and c_d and c_f are the associated costs.

Depending on the query, the cost very much differs, in that for 20% selectivity, filtering first is faster, while for 50% selectivity, doing the NNS first is faster.

Since the distance computation and associated access costs are relative to the vector size, we could use Principal Component Analysis (PCA) or others to reduce the dimension of the vectors. Alternatively, we can quantize some values, with the drawback of reducing precision. Trees offer more residualization, so the best of both worlds is to score fast with quantized vectors, then re-score with the full precision on just a subset.

A further optimization is to change how much of the set we re-score vs how much of it we score.

Not all dataset and query combinations are equally easy, and filters also change the difficulty.

8.5.3 Quantization

The total storage usage of course is always $N \cdot c$, where N is the number of vectors and c is the cost per vector, computed as follows:

- **Scalar-Quantization (SQ)**: $c = i + d \cdot m$, where d is the number of dimensions of the vector and m is the byte count per dim.
- **Product Quantization (PQ)**: $c = i + q$, with i the cost of the vector ID and $q = m \lceil \log_2(k)/8 \rceil$ bytes, with m the number of subvectors, and k the codebook entry count. Remember that we divide by 8 to get bytes, as the $\log_2(k)$ gives us the number of bits.

8.5.4 Filter agnosticism

Filter agnosticism isn't just a single type, it's a spectrum.

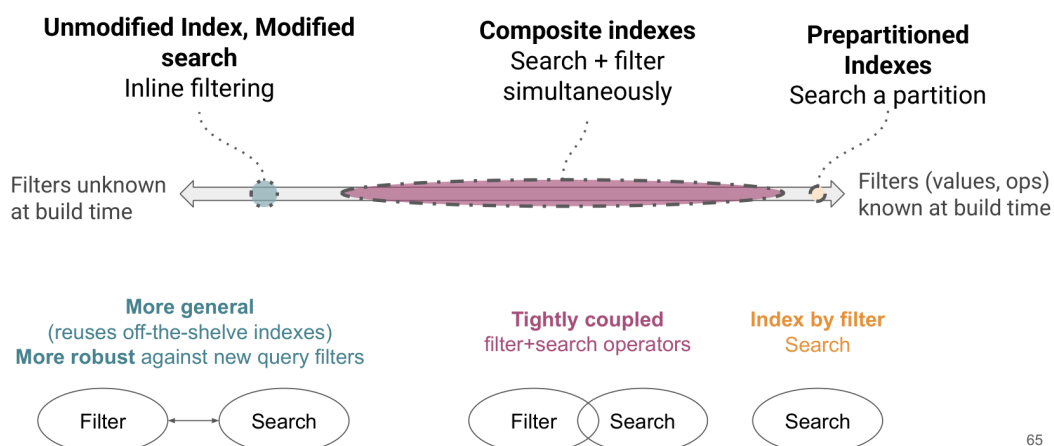


Figure 8.1: Spectrum of filter agnosticism (Figure from lecture slides 18, slide 65)

Ideally, we would build an index per filter, but there are issues:

- Footprint is too high because of duplicated nodes
- Not enough data to build an index on the small partitions

We can emulate this partitioning with efficient footprint using a monolithic graph with pruning.

Definition 8.5.2 Value-induced neighbourhood We can build a new index based on attributes and embedding similarity (embedding distance)

Definition 8.5.3 Predicate Traversal Alternatively we can reuse unfiltered indexes and use inline filtering to discover filter-passing nodes, then light up the right neighbourhood at search time.

Definition 8.5.4 Densified Predicate Traversal We also add edges to alleviate connectivity issues and proceed as with the predicate traversal

9 Data

9.1 Formats

Data formats heavily influence how efficient a database system is. For a 1GB data, JSON is about three times larger than necessary due to field names and text encoding. For the TPC-H lineitem table, the CSV file is 15.95GB, whereas Parquet Snappy and ZSTD are 3.78 and 3.22 Gigabytes respectively, so factor 5 smaller. Furthermore, a raw CSV scanner is about three times slower than a Parquet Scanner.

If we scale that up to Terabytes or more, compute already is dominated by I/O 80-90% of the time, so it really is crucial that we have an efficient format.

9.1.1 Encodings

Typical text encodings are CSV, XML⁴ and JSON. Of course, there are also the (in my objective opinion) best formats⁵ like conf and yaml, as well as TOML, INI and more.

These are good for being human readable, but not if you have terabytes of data. From a practical example, I know that a 1GB JSON file in Python's built in parser takes about 1s to load.

This is why we need (direct) binary encodings for the data, such as the Apache formats.

Examples include Avro (row-oriented format, including a schema describing the fields and types, this is the header of the file), Parquet (Compressed column store), CarbonData (Compressed column store with indexes) and more.

When designing formats, decisions need to be made on the file metadata, format layout, type system, encoding schemes, compression, filters and how to nest the data.

Metadata Ideally the files are self-contained to allow for portability, and for space efficiency reasons (even though it may be beneficial for other reasons), schemas are only stored once per file, in either the header (e.g. for Avro) or in the footer (e.g. Parquet). Other metadata however is more often stored once per frame.

The **format layout** is the physical organization of the bytes on the disk. We can have row stores, column stores or hybrids:

- N-ary Storage Model (NSM): All attributes stored contiguously in a single page (Row store)
- Decomposition Storage Model (DSM): Store single attributes for all tuples contiguously in a block of data (Column store)
- PAX: Horizontally partition data into row groups, then vertically partition the attributes into column chunks.

For the **type system**, we choose what kind of types we want to support, be it primitives such as numbers and strings, or more complex types such as arrays. Furthermore, we choose how they are physically represented, such as IEEE-754 encoding for floating point numbers, and logically, if they are complex types based on other types, i.e. how they abstract a more primitive type.

For **encoding schemes** include:

- *Dictionary encoding*: We find all unique items of a column and build a dictionary with integer keys and then only store the integers. This is ideal for columns with few distinct values and worse than not doing anything for columns with the UNIQUE constraint set.
- *Run-Length encoding*: If we have many repeating values, called runs, we replace each run with a pair (count, value). This can be done on the data itself, or on the binary representation of said data.
- *Delta Encoding*: Store the first value (as-is), then store the difference between this value and the next. Since these deltas are often small, we can fit them into fewer bits.

For **Block compression**, we can use typical compression algorithms, such as LZ4 (which by the way is VERY impressive for compression ratio), Snappy, Zstd (which pacman uses for the packages (.pkg.tar.zst files)), or older algorithms such as gzip or xz (if you remember the whole malware debacle they had, you may not want to use that anymore).

Filters in the files are primarily used to improve traverse performance, by filtering out pages that simply don't need to be searched, using some metadata. For that, statistics are stored in the metadata, such as the min/max, null count, cardinality and more.

⁴you have to be quite a bit of a moron to think that this is the best of the text encodings, but all right

⁵best for configuration files, not (necessarily) in general

9.2 Cloud systems

Cloud systems need to be very reliable, no data is allowed to be lost. In basic versions of so-called **shared nothing instances**, this is not as much guaranteed as it should be, because all DBs systems have their own disks and they don't replicate or exchange their data.

To improve fault tolerance, make blocks small, like 1MB and replicate each block on a different compute node and ideally on a separate S3 node, which serves as a backup. That's the 3-2 taken care of of the 3-2-1 backup strategy. This should then also be replicated off-site to mitigate the risk of destruction of all nodes from a natural disaster, building collapse, fire or similar.

Modern clouds separate compute and storage entirely. Typically, compute servers have no disk and are booted using iSCSI, or they have a small disk for the base operating system in them.

In systems with disaggregated Compute, Memory and Storage, each layer can scale independently, allowing large memory shuffles to take place and allows for larger numbers of blocks for sorting and joins, which, as we have seen, can massively boost performance.

The Big Query system is one such example.

9.3 Key Value Store

The rise of the internet has shown that there is the need for a system that can more quickly retrieve data than the older implementations using blocks, files and objects. The goal is to be able to access a chunk of data that does not need to be parsed (unlike objects), but can be of various sizes (unlike blocks) and needs no system-supported names (like file systems).

The solution to this problem is called a *Key Value Store* (KVS).

Most **SQL** databases follow the **ACID** concept to maintain consistency, etc. The **ACID** concept gives very strong guarantees, thus the work is done by the database engine as opposed to the application. However, it is expensive, both in throughput and latency and does not scale well.

Key-Value stores get rid of EVERYTHING. They are schema-less, **NoSQL**, no **ACID**, (typically) **BASE** based stores. The most well known examples of **NoSQL** DBs are Redis and MongoDB, with Redis being a true Key-Value Store (however usually referred to as a Key-Value Cache).

9.3.1 Architecture

- Data is stored as a key-value pair. The key is used to locate and retrieve the data. The value is a binary object of arbitrary size
- The data is organized as a distributed hash table. We hash the key to get the location for the value
- KV pairs stored in several places for availability
- Access is **quorum**-based for consistency
- Typically eventual consistency
- The physical storage uses already reserved buckets of different sizes and objects are located in the corresponding buckets.
- This can be run in main memory for very fast access, with larger blobs stored in storage.

9.3.2 Advantages and Disadvantages

Advantages

- Very fast lookups (due to hashing)
- Easy to scale to many machines (simply add more copies / machines). This is great for the cloud's elasticity
- Can be implemented in main memory of machines (giving even faster lookups)
- Useful in many applications (looking up user profiles, retrieving user's web pages and info)
- Scalable and easy to get going (no schema needed)

Disadvantages

- No easy support for queries beyond simple key requests
- this means that there are no joins, no ranges, no queries on attributes, etc (because there are none)
- Depending on implementation, data can be inconsistent
- Maintaining consistency is up to the application
- Used in large scale deployments (worldwide), so keeping data in sync can be challenging

As mentioned above, one major drawback of **NoSQL** DBs is that they move the complexity to the application. This means that some operations are very costly to implement (range queries, joins, etc), as they require reading all data from the DB.

9.3.3 Uses of KVS

As a result of the above disadvantages, KVS are typically used as a cache or specialized system and not as a replacement for **SQL** databases.

They are often used as a main memory cache to avoid having to access the cloud storage, resulting in higher throughput and lower latency.

This is especially useful for commonly run queries, such as loading all videos of a commonly accessed YouTube Channel. In that case especially it is easy to invalidate the cache line and updating it, as updates are very rare and they can trigger a cache invalidation for that channel.

9.3.4 Distributed KVS

How do we distribute a KVS across multiple systems? A simple strategy would be to assign data to machines using modulo n for n machines on the hash. In theory that is not a bad idea, but what if we remove one machine? Or if we add one? Then we have to move ALL of the data. If one machine fails, in the worst case, all data could possibly need to be re-hashed, possibly moving ALL of the data to different machines.

A better solution is to use horizontal partitioning of the data using hashing. Each machine deals with the data points in the clockwise direction before the next machine. In other words, each machine deals with the data in a certain range of hashes, the next machine deals with the subsequent ones, etc.

Thus, on fail, or exit in any other way, only the machine after the failed / exited machine needs to modify the data. When adding nodes, the next machine hands some of the responsibilities over to the newly inserted machine.

To account for possible failure of machines, we need to keep multiple copies over multiple machines (ahead in counter-clockwise orientation) to prevent data loss.

9.4 Cloud-Native Databases

Many of the traditional database engines may run into overhead due to heavy I/O traffic, especially when mirroring.

As a result, many companies have improved implementations of SQL databases to make them suitable to cloud infrastructure.

Some of the improvements include caching. Micro-partitions in the Snowflake system is a system to facilitate query processing. Each one of these partitions has sizes between 50 and 500 megabytes, and has metadata describing what is inside, which can also be read without reading the entire partition.

Furthermore, Amazon S3, the storage platform, does not support in-place updates, files are replaced in their entirety. This means that old data can easily be recovered.

9.5 The Future

“The IT industry is, above all, an industry”. This is very much true and trends and developments are driven by demand and market. For hardware, the investment is very high for new developments and only pay off at large sale volumes. Hardware moves slowly, software even slower.

Furthermore, the number of foundries has gone down significantly since 2002 (130nm architecture), where today, there are only three bleeding edge foundries left, of which one has over 50% market share in bleeding edge semiconductor manufacturing, TSMC. Samsung and Intel both have lost out on major deals due to a number of issues (Nvidia Ampere was the last major GPU series manufactured by Samsung).

CPUs are low latency (very fast), but low throughput in terms of parallelism, whereas GPUs are the exact opposite.

10 Quick Overview

This section aims to give you the most important things to remember in a very concise manner.

10.1 SQL

DuckDB (and PostgreSQL) features an EXPLAIN and EXPLAIN ANALYZE query, pretty-printing the query plan and pretty-printing the query plan and profiling the query execution, respectively

A few useful commands for debugging and writing queries (for DuckDB and PostgreSQL via `psql` CLI):

DuckDB	PostgreSQL	Description
SHOW TABLES;	\dt	Show all tables
SHOW tablename;	\d tablename	Show schema of table tablename

Note that SHOW and DESCRIBE are aliases in DuckDB. They can also be used on queries!.

To see all `psql` commands, run `\?`.

10.2 System

For computing the usable space, deduct the overhead *after* multiplying with the free fraction, e.g. for a block size of 4kB (with kB actually being KiB), header and directories having a combined size of 100 bytes and PCTFREE set to 20%, the available space is $\lceil 4096 \cdot 0.8 - 100 \rceil = 3177$.

To compute the maximum number of tuples in a slotted page, remember that it also has a header with a certain number of bytes for the slotting. Add that to the number of bytes per tuple when computing, even though it doesn't count to the tuple size!

Row stores are better if we need to access many columns, column stores otherwise. This is (primarily) due to indexing.

10.2.1 Indexing

The global depth is the maximum Local Depth of all buckets.

The hash key is the actual value of the column to be hashed typically. Thus, an assignment may be very misleading in that the block ID and hash key are asked. The hash key is the value before hashing, the block ID afterwards is the block the hashing algorithm assigned it to.

When splitting a block, its N entries must be repartitioned. There are 2^N possible ways in which to do this. So, there is a $\frac{2}{2^N}$ probability for there to be a **recursive overflow**, an overflow in which all keys end up in the same bucket again and need further splitting.

10.3 Query Processing

10.3.1 Sorting

Given B frames of memory and N records, we have

- **Merge Sort:** $2N \cdot P$, with $P = (1 + \lceil \log_{B-1} \lceil N \div B \rceil \rceil)$ the number of passes. After the first pass, $\lceil N \div B \rceil$ number of sorted runs were created (typically)
- **Replacement Sort / Heap Sort:** $2N \cdot P$, with $P = (1 + \lceil \log_{B-1} \lceil N \div E \rceil \rceil)$ the number of passes, with E the number of records per run, given by $E = 2B$ in the average case, $E = N$ in the best case (thus single run, data already sorted), $B - 1$ (worst case, reversed data)

10.3.2 Selection

- **File Scan:** $N \div P_F$, with P_F the number of pages, because we need to run through all pages
- **B+ Tree:** $\text{height}(T) + \text{cnt}(P_L) + \text{cnt}(P_F)$, P_L the leaf pages and P_F the file pages.
 - For a clustered index, $\text{cnt}(P_F) = \text{cnt}(R') \div (F_L \cdot R_L)$, with R_L the number of records per leaf page and F_L the fill factor.
 - For an unclustered index, $\text{cnt}(P_F) = \text{cnt}(R')$, with $\text{cnt}(R') = \text{cnt}(R) \cdot S$, with S the selectivity of the predicate.
- **Hash Index:** Cost is $1 + \text{cnt}(R') \div \text{cnt}(P_L) + \text{cnt}(R')$, where the 1 is to do the lookup, $\text{cnt}(R') \div P_L$ to fetch the row IDs and $\text{cnt}(R')$ to fetch the records (this is an up-to, it is $\text{cnt}(R') \div P_F$ as minimum, for clustered index), with P_L the number of records per Leaf and P_F the number of records per file.

10.3.3 Projection

- **Partitioning:** $\text{Cost}_{\text{part}}(R) = \text{cnt}(R) + \text{cnt}(R')$ with $\text{cnt}(R') = \text{cnt}(R)Q$, with Q the fraction of selected attributes divided by total attributes
- **Sort-Based:** The number of sorted runs are $M = \lceil \text{cnt}(R') \div B \rceil$, with the merge passes $P = \lceil \log_{B-1}(M) \rceil$, total cost: $2 \cdot N \cdot P + \text{Cost}_{\text{part}}(R)$
- **Hash-Based:**
 - **Minimum B for M -pass:** $B > \sqrt{\text{cnt}(R') \cdot f}$, with f the Minister of Magic Factor (for Books ≤ 5), aka. Fudge Factor, typically $f \approx 1.2$.
 - **Below threshold:** To check if we need recursive partitioning, we use the un-approximated version of the above: $\frac{f \cdot \text{cnt}(R')}{B-1} < B$
 - **Cost:** The base cost is $\text{Cost}_{\text{part}}(R) + \text{cnt}(R')$ for partitioning and duplicate elimination later on
 - * If below threshold, that's our cost
 - * If above threshold, then we need to repartition, which costs $2 \cdot \text{cnt}(R')$ (once each for write and read) for each time we do this action.

Note that both the sort- and hash-based approaches perform approximately equally well at larger B , since both algorithms need two passes over the data.

10.4 Checklist

The following things are typically important to know very well (not exhaustive)

- ☐ SQL (syntax, some concepts, such as using WITH statements often to make the task more approachable)
- ☐ Functional Dependencies (including Candidate Keys, Super Keys, Closures and minimal covers)
- ☐ Normal Forms (and their Decomposition / Synthesis Algorithms)
- ☐ Time complexities, I/Os and sorted runs for Query Processing
- ☐ Conflict Serializability
- ☐ Core concepts of Vector Search
- ☐ Recoverability (both the normal techniques, plus Snapshot Isolation and 2-Phase Locking (and strict variant thereof))

Glossary

ACID Atomicity, Consistency, Isolation, Durability, also see Section 7.2.

ANN Approximate Nearest Neighbour Search.

AVX The Advanced Vector Extensions are an extension to the x86 architecture, to allow CPUs to execute SIMD instruction (commonly called Vector Instructions).

AVX Advanced Vector Extensions, also see **AVX**.

bag A bag, or multiset, is a set, in which, (as is usual with sets) order does not matter, but duplicates are allowed.

BASE Basically Available, Soft State, Eventually Consistent.

DBMS Database Management System.

declarative Describes *what* to achieve, but not how.

imperative Describes how to achieve something.

KNN K -Nearest Neighbour Search.

NoSQL SQL-less database (commonly a Key-Value Store, see Section 9.3).

quorum See this Wikipedia article. TL;DR: It is a synchronization system for distributed computing. .

RA Relational Algebra, see Section 2.2.

record Typically a row or tuple in the database.

schema A set of relations, defining the attributes of the tuples.

SIMD Single Instruction Multiple Data.

SQL Structured Query Language.

subquery Also Nested Query, a separate query that can be used in the FROM or WHERE clause of another query. See 3.3.2.1.

superkey See Section 4.1.